

1. Given the quadratic function in standard form, address the following.

$$f(x) = -(x-3)^2 - 4$$

- a. What are the coordinates of the vertex?

The vertex is $(3, -4)$.
(Type an ordered pair.)

- b. Does the graph "open up" or "open down"?

opens down

opens up

- c. What is the equation of the axis of symmetry?

The axis of symmetry is $x=3$.
(Type an equation.)

- d. Find any x-intercepts. Select the correct choice below and, if necessary, fill in the answer box to complete your choice.

A. $x =$

(Type an exact answer, using radicals as needed. Use a comma to separate

B. There is no x-intercept.

- e. Find the y-intercept. Select the correct choice below and, if necessary, fill in the answer box to complete your choice.

A. $y = -13$

(Type an integer or a fraction.)

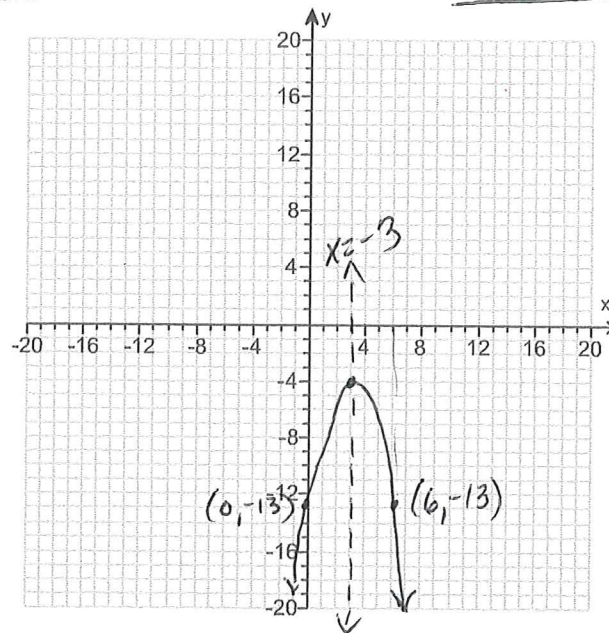
B. There is no y-intercept.

- f. Sketch the graph. Use the graphing tool to graph the function.

- g. State the domain and range.

The domain of f is the interval $(-\infty, \infty)$.
(Type your answer in interval notation.)

The range of f is the interval $(-\infty, -4]$.
(Type your answer in interval notation.)



x-intercepts: $(y=0)$

$$0 = -(x-3)^2 - 4$$

$$4 = -(x-3)^2$$

$$\sqrt{-4} = \sqrt{(x-3)^2}$$

↑ no real nos for x-intercepts

y-intercepts $(x=0)$

$$f(0) = -(0-3)^2 - 4$$

$$= -9 - 4$$

$$= -13$$

2. Given the quadratic function in standard form, address the following.

a. Sketch the graph.

b. State the domain and range. *Vertex: $(-5, -9)$*

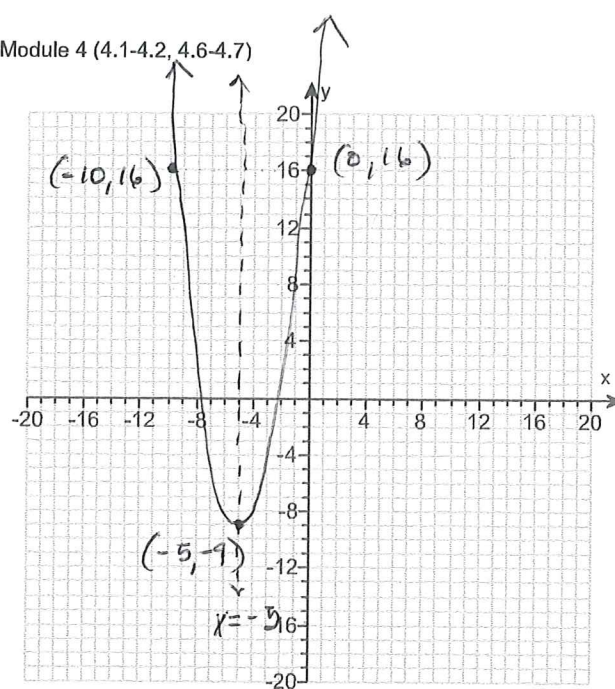
$$f(x) = (x + 5)^2 - 9$$

a. Use the graphing tool to graph the function.

b. The domain of f is the interval $(-\infty, \infty)$.
(Type your answer in interval notation.)

The range of f is the interval $[-9, \infty)$.
(Type your answer in interval notation.)

Test #4, Module 4 (4.1-4.2, 4.6-4.7)



3. First rewrite the given quadratic function in standard form by completing the square, then address the following.

$$f(x) = -1(x^2 + 2x + 1) + 15 - (-1)$$

$$= -1(x+1)^2 + 16$$

$$f(x) = -x^2 - 2x + 15$$

Rewrite the quadratic function in standard form.

$$f(x) = -1(x+1)^2 + 16$$

- a. What are the coordinates of the vertex?

The vertex is $(-1, 16)$.
(Type an ordered pair.)

y-intercept: $(0, 15)$
x-intercepts: $(3, 0)$ & $(-5, 0)$

- b. Does the graph "open up" or "open down"?

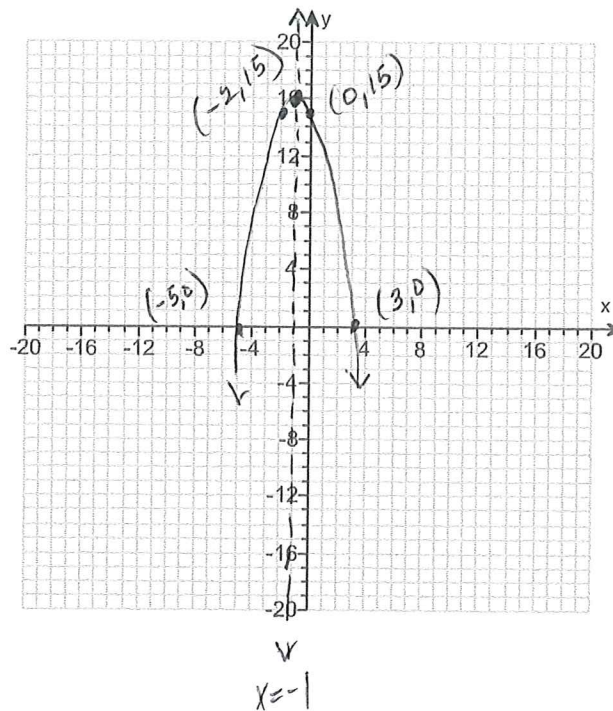
opens down

opens up

- c. What is the equation of the axis of symmetry?

The axis of symmetry is $x = -1$.
(Type an equation.)

- d. Sketch the graph. Use the graphing tool to graph the function.



4. A toy rocket is shot vertically into the air from a 5-foot-tall launching pad with an initial velocity of 168 feet per second. Suppose the height of the rocket in feet t seconds after being launched can be modeled by the function $h(t) = -16t^2 + v_0t + h_0$, where v_0 is the initial velocity of the rocket and h_0 is the initial height of the rocket. How long will it take for the rocket to reach its maximum height? What is the maximum height?

$$h(t) = -16t^2 + 168t + 5$$

The rocket will reach its maximum height in 5.25 second(s).

$$t = \frac{-168}{2(-16)} = \frac{21}{4} = 5.25 \text{ secs}$$

The maximum height reached by the rocket is 446 feet.

$$h(5.25) = -16(5.25)^2 + 168(5.25) + 5 = 446 \text{ ft.}$$

5. A farmer has 2500 feet of fencing available to enclose a rectangular area bordering a river. If no fencing is required along the river, find the dimensions of the fence that will maximize the area. What is the maximum area?

Find the dimensions of the fence that will maximize the area.

$$P = 2W + L \quad A = W \cdot L \quad 2500 = 2W + L$$

$$A = W(2500 - 2W) \quad 2500 - 2W = L$$

$$A = 2500W - 2W^2 \quad L = 2500 - 2(625)$$

$$W = \frac{-2500}{2(-2)} = 625' \quad L = 1250'$$

Width = 625 feet
Length = 1,250 feet

The maximum area is 781,250 square feet.

6. Find all vertical asymptotes and create a rough sketch of the graph near each asymptote.

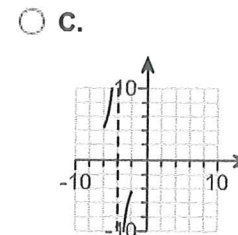
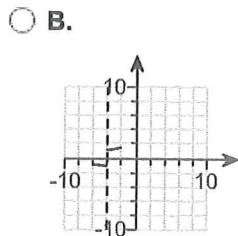
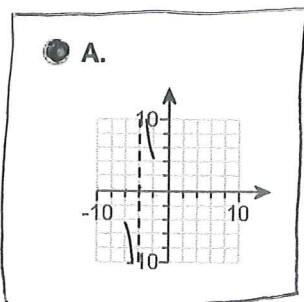
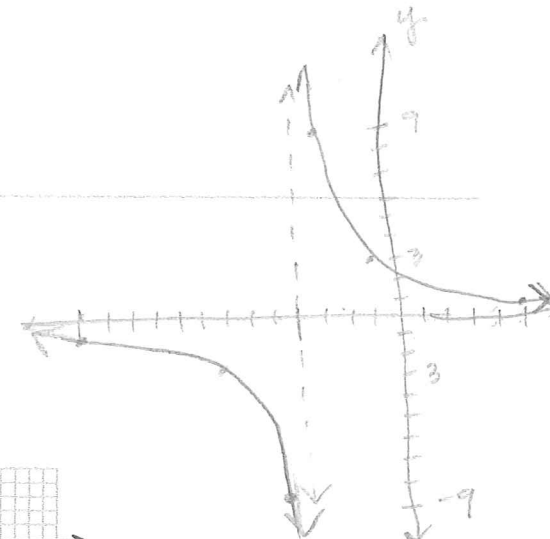
$$f(x) = \frac{9}{x+4}$$

There is a vertical asymptote at $x = -4$.

(Type an integer or a simplified fraction. Use a comma to separate answers.)

Which graph best represents the behavior of $f(x)$ near the vertical asymptote(s)? Choose the correct graph below.

X	Y
-3	9
-1	3
5	1
-5	-9
-7	-3
-13	-1



7. Find all vertical asymptotes.

$$f(x) = \frac{x+9}{x^2-5x+4}$$

$$x^2 - 5x + 4 = 0$$

$$(x-4)(x-1) = 0$$

$$x = 4, x = 1$$

There is a vertical asymptote at $x = 1, 4$.

(Type an integer or a simplified fraction. Use a comma to separate answers.)

8. Find the equation of all horizontal asymptotes (if any) of the rational function.

$$f(x) = \frac{3}{x-4}$$

Select the correct choice below and fill in any answer boxes within your choice.

A. There is a horizontal asymptote at $y = 0$.

(Type an integer or a simplified fraction. Use a comma to separate answers.)

B. There is no horizontal asymptote.

9. Find the equation of all horizontal asymptotes (if any) of the rational function.

$$f(x) = \frac{x^2 - 9}{x - 3}$$

Select the correct choice below and fill in any answer boxes within your choice.

- A. There is a horizontal asymptote at $y =$.
(Type an integer or a simplified fraction. Use a comma to separate answers.)
- B. There is no horizontal asymptote.

10. Find the equation of all horizontal asymptotes (if any) of the rational function.

$$f(x) = \frac{9x^2}{8 - 3x^2}$$

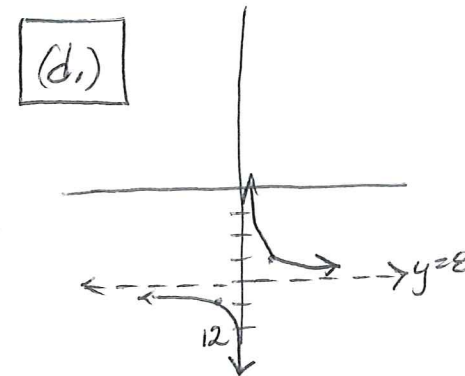
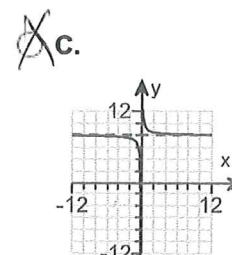
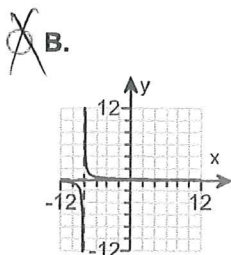
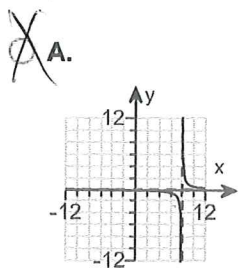
Select the correct choice below and fill in any answer boxes within your choice.

- A. There is a horizontal asymptote at $y =$.
(Type an integer or a simplified fraction. Use a comma to separate answers.)
- B. There is no horizontal asymptote.

11. Use transformations of $y = \frac{1}{x}$ or $y = \frac{1}{x^2}$ to sketch the rational function. Label all intercepts and find the equations of all asymptotes.

$$f(x) = \frac{1}{x} - 8$$

Graph the transformation. Choose the correct graph below.



Identify the x-intercept(s). Select the correct choice below and fill in any answer boxes within your choice.

$$\begin{aligned} \frac{1}{x} - 8 &= 0 \\ \frac{1}{x} &= 8 \\ 8x &= 1 \\ x &= \frac{1}{8} \end{aligned}$$

- A. $x = \frac{1}{8}$
 (Type an integer or a simplified fraction. Use a comma to separate answers.)
- B. There is no x-intercept.

Identify the y-intercept. Select the correct choice below and fill in any answer boxes within your choice.

- A. $y =$ _____
 (Type an integer or a simplified fraction. Use a comma to separate answers.)
- B. There is no y-intercept.

Identify the horizontal asymptote(s). Select the correct choice below and fill in any answer boxes within your choice.

- A. $y = -8$
 (Type an integer or a simplified fraction. Use a comma to separate answers.)
- B. There is no horizontal asymptote.

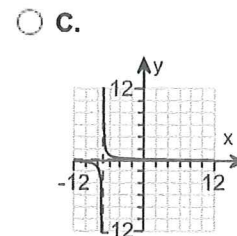
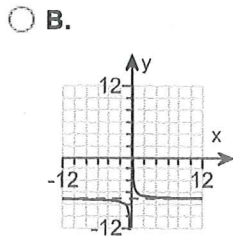
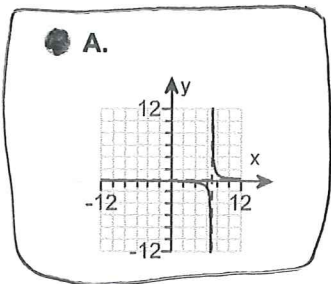
Identify the vertical asymptote(s). Select the correct choice below and fill in any answer boxes within your choice.

- A. $x = 0$
 (Type an integer or a simplified fraction. Use a comma to separate answers.)
- B. There is no vertical asymptote.

12. Use transformations of $y = \frac{1}{x}$ or $y = \frac{1}{x^2}$ to sketch the rational function. Label all intercepts and find the equations of all asymptotes.

$$f(x) = \frac{1}{x-7}$$

Graph the transformation. Choose the correct graph below.



Identify the x-intercept(s). Select the correct choice below and fill in any answer boxes within your choice.

- A. $x =$ (Type an integer or a simplified fraction. Use a comma to separate answers as needed.)

B. There is no x-intercept.

$$\frac{1}{x-7} = \frac{0}{1} \quad 1 \neq 0 \rightarrow \text{no } x\text{-intercepts}$$

Identify the y-intercept. Select the correct choice below and fill in any answer boxes within your choice.

- A. $y = -\frac{1}{7}$ (Type an integer or a simplified fraction. Use a comma to separate answers as needed.)

B. There is no y-intercept.

$$\frac{1}{0-7} = -\frac{1}{7}$$

Identify the horizontal asymptote(s). Select the correct choice below and fill in any answer boxes within your choice.

- A. $y = 0$ (Type an integer or a simplified fraction. Use a comma to separate answers as needed.)

B. There is no horizontal asymptote.

Identify the vertical asymptote(s). Select the correct choice below and fill in any answer boxes within your choice.

- A. $x = 7$ (Type an integer or a simplified fraction. Use a comma to separate answers as needed.)

B. There is no vertical asymptote.

13. Follow the nine-step graphing strategy to sketch the graph of the rational function.

$$f(x) = \frac{6x + 9}{x - 2}$$

1. Find the domain.

The domain of f is $(-\infty, 2) \cup (2, \infty)$

(Type your answer in interval notation. Use integers or fractions for any numbers in the expression.)

2. Does f have any removable discontinuities? Select the correct choice below and, if necessary, fill in the answer boxes to complete your choice.

A. There is a removable discontinuity at _____ . (Type an ordered pair.)

The function simplifies to $f(x) =$ _____ .

B. There are no removable discontinuities.

3. Find the y-intercept. Select the correct choice below and, if necessary, fill in the answer box to complete your choice.

A. The y-intercept is $y = -\frac{9}{2}$. (Simplify your answer.)

B. The function has no y-intercept.

4. Find any x-intercepts. Select the correct choice below and, if necessary, fill in the answer box to complete your choice.

A. The x-intercept(s) is(are) $x = -\frac{3}{2}$.
(Simplify your answer. Use a comma to separate answers as needed.)

$$\begin{aligned} 6x + 9 &= 0 \\ 6x &= -9 \\ x &= -\frac{9}{6} = -\frac{3}{2} \end{aligned}$$

B. The function has no x-intercept.

5. Find the equations of any vertical asymptotes. Select the correct choice below and, if necessary, fill in the answer box to complete your choice.

A. The vertical asymptote(s) is(are) $x = 2$.
(Type an equation. Use a comma to separate answers as needed.)

B. The function has no vertical asymptotes.

6. Determine whether the graph has a horizontal asymptote or a slant asymptote. Select the correct choice below and, if necessary, fill in the answer box to complete your choice.

A. The horizontal asymptote is $y = 6$. (Type an equation.)

B. The function has no horizontal asymptote. It has a slant asymptote of _____ .
(Type an equation. Type your answer in slope-intercept form.)

C. The function has no horizontal asymptote and has no slant asymptote.

$$(-3, f(-3)) = (-3, \frac{9}{5})$$

$$(0, f(0)) = (0, -\frac{9}{2})$$

$$(3, f(3)) = (3, \frac{27}{2})$$

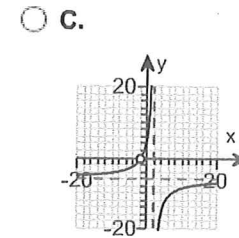
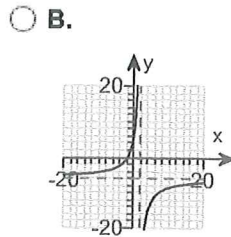
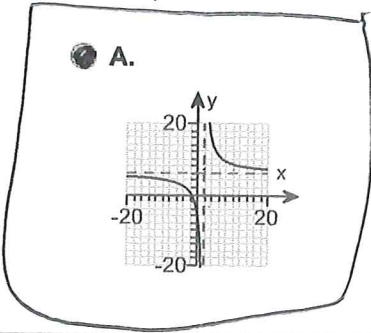
$$\frac{6(-3)+9}{-3-2} = \frac{-9}{-5}$$

$$\frac{6(0)+9}{0-2} = -\frac{9}{2}$$

$$\frac{6(3)+9}{3-2} = \frac{27}{1}$$

(Simplify your answers. Type integers or fractions.)

8. Complete the sketch. Choose the correct graph below.



*14. List all the asymptotes of the following function, then graph the function.

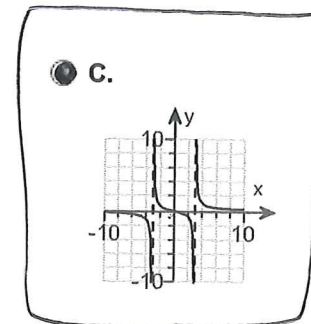
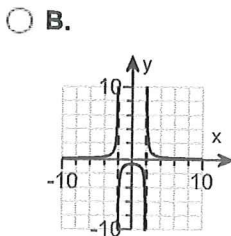
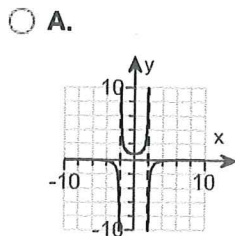
$$f(x) = \frac{2x}{x^2 - 9}$$

VA: $x = \pm 3$ HA: $y = 0$

Choose the correct asymptotes below.

- A. $x = 2, x = -2, y = 0$
- B. $x = 2, x = -2$
- C. $x = 3, x = -3, y = 0$
- D. $x = 3, x = -3$

Choose the correct graph below.



x	y
1	-1/4
0	0
-1	1/4
4	8/7
5	5/8
-4	-8/7
-5	-5/8

middle
right
left

*15. Find the variation constant and an equation of variation where y varies directly as x and y = 3 when x = 1.

The variation constant is $k = 3$.

The equation of variation is $y = 3x$.

$$y = kx$$

$$3 = k \cdot 1$$

$$3 = k$$

*16. Find the variation constant and an equation of variation where y varies inversely as x and y = 19 when x = 11.

The variation constant is 209.

The equation of variation is $y = \frac{209}{x}$.

$$y = \frac{k}{x}$$

$$19 = \frac{k}{11}$$

$$209 = k$$

*17. Find an equation of variation in which y varies jointly as x and z, and y = 10 when x = 5 and z = 2.

The equation of variation is $y = kxz$. (Simplify your answer.)

$$y = kxz$$

$$10 = k \cdot 5 \cdot 2$$

$$10 = 10k$$

$$1 = k$$

*18. Find an equation of variation in which y varies jointly as x and z and inversely as the product of w and p, where $y = \frac{7}{12}$ when x = 7, z = 14, w = 3, and p = 8.

The equation of variation is $y = \frac{kxz}{wp}$. (Simplify your answer. Type an integer or a fraction.)

$$y = \frac{kxz}{wp}$$

$$\frac{7}{12} = \frac{k(7)(14)}{(3)(8)4}$$

$$\frac{7}{12} = \frac{49k}{12}$$

$$7 = 49k$$

$$\frac{1}{7} = k$$

*19. Write an equation that expresses the relationship. Then solve the equation for b.

f varies jointly as b and the square of q.

$f = kbq^2$
(Use k as the constant of variation.)

Solve for b.

$$b = \frac{f}{kq^2}$$

20. The ^(y)value of a car is inversely proportional to its ^(x)age. If a car is worth \$4900 when it is 3 years old, how old will it be when it is worth \$2100?

The car will be 7 years old.

$$y = \frac{k}{x}$$

$$\frac{4900}{3} = \frac{k}{1}$$

$$k = 14,700$$

$$\frac{14,700}{x} = \frac{2100}{1}$$

$$2100x = 14,700$$

$x = 7$ yrs
old

21. Graph the following quadratic function on your ~~test~~ paper. Sketch the axis of symmetry as a dashed line. Use a minimum of 5 points including the vertex.

$$f(x) = 2x^2 + 4x - 1$$

$$f(x) = 2(x^2 + 2x + 1) - 1 - 2$$

$$= 2(x+1)^2 - 3$$

Vertex: (-1, -3)

Sketch on graph paper.

(see last page)

22. Graph the following rational function on your ~~test~~ paper. Sketch all asymptotes as dashed lines. Use a minimum of ~~five~~ ⁶ points.

$$f(x) = \frac{2x}{x-3}$$

VA: $x = 3$
HA: $y = 2$

Sketch on graph paper.

(see last page)

23. Graph the rational function on your ~~test~~ paper. Sketch all asymptotes as dashed lines. Use a minimum of five points. *sketch on graph paper.*

$$f(x) = \frac{2}{x^2 - 10x + 21}$$

(see below)

$$\begin{aligned} x^2 - 10x + 21 &= 0 \\ (x-7)(x-3) &= 0 \\ x &= 7, 3 \rightarrow \text{VAs} \end{aligned}$$

HA: $y = 0$

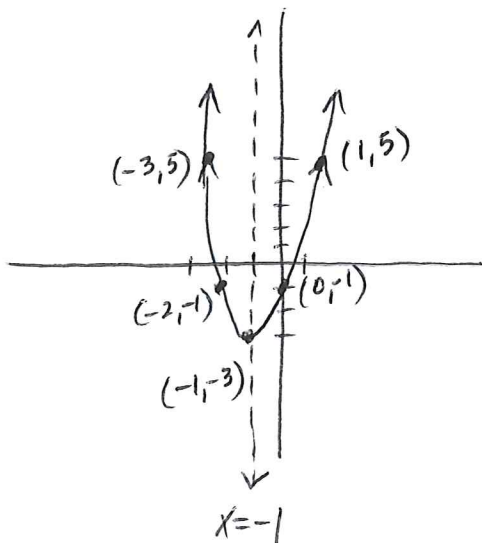
$$y = \frac{2}{(x-7)(x-3)}$$

21) $f(x) = 2x^2 + 4x - 1$

Vertex: $(-1, -3)$

axis of sym: $x = -1$

x	y
-1	-3
0	-1
-2	-1
1	5
-3	5

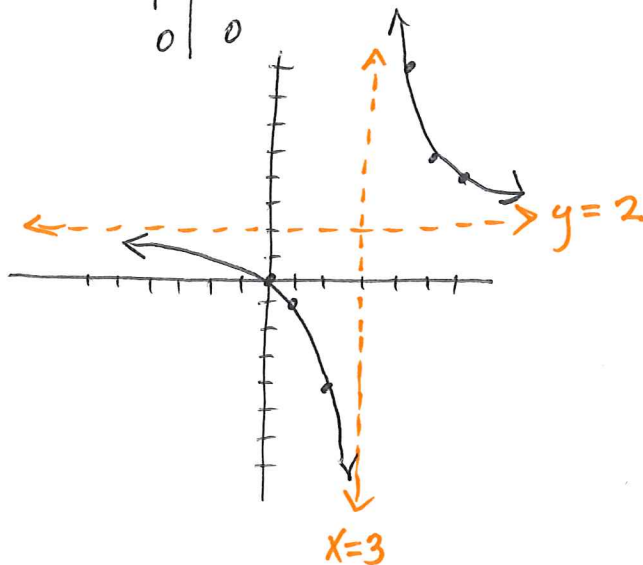


22) $f(x) = \frac{2x}{x-3}$

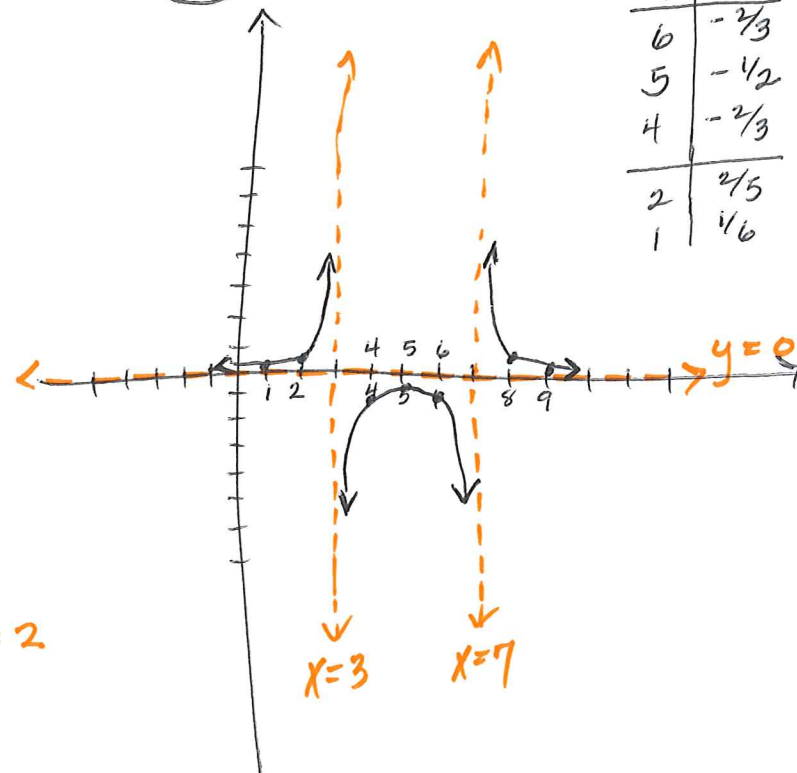
VA: $x = 3$

HA: $y = 2$

x	y
4	8
5	5 (right)
6	4
2	-4 (left)
1	-1
0	0



23



x	y
8	2/5
9	1/6
6	-2/3
5	-1/2
4	-2/3
2	2/5
1	1/6