MAC 1140 (Important theorems in sections 3.2 and 3.3)

The Remainder Theorem

When a polynomial $f(x)$ is divided by x – c, then the remainder is f (c).

Example: $f(x) = 2x^3 - 3x^2 + 4$

You can find f (3) by dividing $f(x)$ by (x – 3):

3	2	-3	0	4
		6	9	27
	2	3	9	**31**

f (3) = **31** ↑ remainder

The Factor Theorem

(x – c) is a factor of $f(x)$ $\Longleftrightarrow$ c is a zero of $f(x)$

Example: If a function has factors: (x – 5), (x – 2), and (x + 3),
then the function has zeros: 5, 2, and –3

Likewise, if a function has zeros: 4, -2, and 3 ← mult. 2,
then the function has factors: (x – 4)(x – (-2))(x – 3)(x – 3)
$= (x-4)(x+2)(x-3)^2$

The Fundamental Theorem of Algebra

Every polynomial equation (of degree one or higher) has at least one solution.
(In other words, you can't write a polynomial equation that doesn't have a solution.)

Example: These are guaranteed to have a solution because they are
polynomial equations: $3x^4 - 5x^3 + 2x - 5 = 0$, $5x^3 - \frac{2}{3}x + 8 = 0$

These are **not** guaranteed to have a solution because they are
not polynomial equations: $\frac{3}{x} + 5 = 0$, $2\log(3x) = 0$

Linear Factors Theorem
A polynomial function, of degree *n*, where $n \geq 1$, can be factored as the product of *n* linear factors.

Example: $g(x) = 2x^4 + 5x^3 + 4x^2 + 5x + 2$ $\leftarrow$ a 4th degree function

$$= (x+2)(2x^3 + x^2 + 2x + 1)$$
$$= (x+2)\left(x^2(2x+1) + 1(2x+1)\right)$$
$$= (x+2)(2x+1)(x^2+1)$$
$$= (x+2)(2x+1)(x-i)(x+i) \quad \leftarrow 4 \text{ linear factors}$$

***n* – Root Theorem**
Every polynomial equation of degree *n*, where $n \geq 1$, has exactly *n* roots.
(A root of multiplicity *k* is counted *k* times.)

Conjugate Pairs Theorem
If $a + bi$ is a solution of a polynomial equation, then its conjugate, $a - bi$, is also a solution of the equation. (In other words, imaginary solutions come in conjugate pairs.)

Theorem
If $a + b\sqrt{c}$ is a solution of a polynomial equation (which has rational coefficients), then its conjugate, $a - b\sqrt{c}$, is also a solution of the equation.

*note: This <u>only</u> applies to solutions of the form $a \pm b\sqrt{c}$, not to solutions of the form $a\sqrt[3]{b}$, $a + b\sqrt[4]{c}$, $\sqrt[5]{a}$, etc.
