Calc II Test #3 Study Guide

① p. 524 #3 $\int_0^1 \cos(x^2)$ $(n=4)$ $\Delta x = \frac{1-0}{4} = \frac{1}{4}$

(a) $T_4 = \frac{1}{4}\cdot\frac{1}{2}\left[f(0) + 2f(\tfrac{1}{4}) + 2f(\tfrac{1}{2}) + 2f(\tfrac{3}{4}) + f(1)\right]$

$= .125\left[1 + 2(.99805) + 2(.96891) + 2(.84592) + .5403\right]$

$\boxed{T_4 = .8957575}$

(b) $M_4 = \frac{1}{4}\left[f(\tfrac{1}{8}) + f(\tfrac{3}{8}) + f(\tfrac{5}{8}) + f(\tfrac{7}{8})\right]$

$= .25\left[.99988 + .99013 + .92467 + .72095\right]$

$\boxed{M_4 = .9089075}$

Since the graph of $f(x) = \cos(x^2)$ is cc down, the T_4 is an underestimate and M_4 is an overestimate so...

$.8957575 < \int_0^1 \cos(x^2)\,dx < .9089075$

The calculator gives $\int_0^1 \cos(x^2)\,dx = .90452424.$

② p. 524 #11 $\int_0^4 x^3\sin(x)\,dx$; $(n=8)$ $\Delta x = \frac{4-0}{8} = \frac{1}{2}$

(a) $T_8 = \frac{1}{2}\cdot\frac{1}{2}\left[f(0) + 2f(\tfrac{1}{2}) + 2f(1) + 2f(\tfrac{3}{2}) + 2f(2) + 2f(\tfrac{5}{2}) + 2f(3) + 2f(\tfrac{7}{2}) + f(4)\right]$

$= .25\left[0 + 2(.05993) + 2(.84147) + 2(3.3665) + 2(7.2744) + 2(9.3511) + 2(3.8102) + 2(-15.04) + -48.44\right]$

$\boxed{T_8 = -7.2782}$

$M_8 = \frac{1}{2}\left[f(\tfrac{1}{4}) + f(\tfrac{3}{4}) + f(\tfrac{5}{4}) + f(\tfrac{7}{4}) + f(\tfrac{9}{4}) + f(\tfrac{11}{4}) + f(\tfrac{13}{4}) + f(\tfrac{15}{4})\right]$

$= .5\left[.00387 + .28757 + 1.8535 + 5.2735 + 8.8627 + 7.9374 + -3.714 + -30.14\right]$

$\boxed{M_8 = -4.81773}$

③ p. 534 #11

$$\int_0^\infty \frac{x^2}{\sqrt{1+x^3}}dx = \lim_{t\to\infty}\int_0^t \frac{x^2}{\sqrt{1+x^3}}dx \quad \left[\begin{array}{l} u = 1+x^3 \\ du = 3x^2dx \\ \frac{1}{3}du = x^2dx \end{array}\right]$$

$$\lim_{t\to\infty}\frac{1}{3}\int_0^t \frac{1}{\sqrt{u}}du = \lim_{t\to\infty}\frac{1}{3}\int_0^t u^{-1/2}du = \lim_{t\to\infty}\frac{2}{3}u^{1/2}\Big]_0^t = \lim_{t\to\infty}\frac{2}{3}(1+x^3)^{1/2}$$

$$\frac{2}{3}\lim_{t\to\infty}(1+x^3)^{1/2}\Big]_0^t = \frac{2}{3}\lim_{t\to\infty}\left[(1+t^3)^{1/2}-(1+0^3)^{1/2}\right] =$$

$$\frac{2}{3}\lim_{t\to\infty}t^{3/2} = \infty \quad \therefore \underline{\text{divergent}}.$$

④ p. 534 #17

$$\int_1^\infty \frac{1}{x^2+x}dx = \lim_{t\to\infty}\int_1^t \frac{1}{\underset{x(x+1)}{x^2+x}}dx \quad \text{[partial fractions]}$$

$$\lim_{t\to\infty}\int_1^t\left(\frac{1}{x}-\frac{1}{x+1}\right)dx = \lim_{t\to\infty}\left[\ln(x)-\ln(x+1)\right]_1^t =$$

$$\lim_{t\to\infty}\left[\ln\left(\frac{x}{x+1}\right)\right]_1^t = \lim_{t\to\infty}\left[\ln\left(\frac{t}{t+1}\right)-\ln\left(\frac{1}{2}\right)\right] = \underset{\text{(use log rules)}}{0-\ln\left(\tfrac{1}{2}\right)} = \boxed{\ln(2)}$$

⑤ p. 549 #9

$$y = 1+6x^{3/2}, \quad 0 \le x \le 1$$

$$\frac{dy}{dx} = 9x^{1/2} \quad 1+(9x^{1/2})^2 = 1+81x \quad L = \int_0^1\sqrt{1+81x}\,dx \quad \left[\begin{array}{l} u = 1+81x \\ du = 81\,dx \\ \frac{1}{81}du = dx \end{array}\right]$$

$$L = \frac{1}{81}\int_1^{82} u^{1/2}du = \frac{1}{81}\cdot\frac{2}{3}\left[u^{3/2}\right]_1^{82} = \frac{2}{243}\left[82^{3/2}-1\right] = \boxed{\frac{2}{243}\left[82\sqrt{82}-1\right]}$$

⑥ p. 549 #13

$$x = \frac{1}{3}\sqrt{y}\,(y-3) \quad 1 \le y \le 9$$

$$x = \frac{1}{3}y^{3/2}-y^{1/2} \quad \frac{dx}{dy} = \frac{1}{2}y^{1/2}-\frac{1}{2}y^{-1/2} \quad 1+\left(\frac{1}{2}y^{1/2}-\frac{1}{2}y^{-1/2}\right)^2 =$$

$$1+\frac{1}{4}y-\frac{1}{2}+\frac{1}{4}y^{-1} = \frac{1}{4}y+\frac{1}{2}+\frac{1}{4}y^{-1} = \left(\frac{1}{2}y^{1/2}+\frac{1}{2}y^{-1/2}\right)^2 \ldots$$

$$L = \int_1^9\left(\frac{1}{2}y^{1/2}+\frac{1}{2}y^{-1/2}\right)dy = \frac{1}{2}\left[\frac{2}{3}y^{3/2}+2y^{1/2}\right]_1^9 = \frac{1}{2}\left[\left(\frac{2}{3}\cdot 27+2\cdot 3\right)-\left(\frac{2}{3}+2\right)\right] = \boxed{\frac{32}{3}}$$

(7) p. 555 #11 $y=\cos(\frac{1}{2}x),\ 0\le x\le \pi$ $\quad y'=-\frac{1}{2}(\sin(\frac{1}{2}x))$

$$S=\int_0^{\pi} 2\pi y\sqrt{1+(y')^2}\,dx = 2\pi\int_0^{\pi}\cos(\tfrac{1}{2}x)\sqrt{1+\tfrac{1}{4}\sin^2(\tfrac{1}{2}x)}\,dx$$

$$\left[\begin{array}{l} u=\sin(\frac{1}{2}x) \\ du=\frac{1}{2}\cos(\frac{1}{2}x)\,dx \\ 2du=\cos(\frac{1}{2}x)\,dx \end{array}\right] \quad 2\pi\int_0^1\sqrt{1+\tfrac{1}{4}u^2}\cdot 2du = 2\pi\int_0^1\sqrt{4+u^2}\,du \ \left(\begin{array}{c}\text{rule}\\ 21\end{array}\right)$$

$$2\pi\left[\frac{u}{2}\sqrt{4+u^2}+2\ln(u+\sqrt{4+u^2})\right]_0^1 = 2\pi\left[\left(\tfrac{1}{2}\sqrt{5}+2\ln(1+\sqrt{5})\right)-\left(0+2\ln(2)\right)\right]=$$

$$\boxed{\pi\sqrt{5}+4\pi\ln\left(\frac{1+\sqrt{5}}{2}\right)}$$

(8) p. 556 # 15 $y=\frac{1}{3}x^{3/2},\ 0\le x\le 12$ $\quad y'=\frac{1}{2}x^{1/2} \Rightarrow 1+\left(\frac{1}{2}x^{1/2}\right)^2=$

$1+\frac{1}{4}(x)$

$$S=\int_0^{12} 2\pi x\sqrt{1+1/4x}\,dx = 2\pi\int_0^{12} x\cdot\tfrac{1}{2}\sqrt{4+x}\,dx \quad \left[\begin{array}{l} u=4+x \\ du=dx \\ u-4=x \end{array}\right]$$

$$S=2\cdot\tfrac{1}{2}\pi\int_4^{16}(u-4)u^{1/2}du = \pi\int_4^{16}\left(u^{3/2}-4u^{1/2}\right)du=$$

$$\pi\left[\tfrac{2}{5}u^{5/2}-\tfrac{8}{3}u^{3/2}\right]_4^{16} = \pi\left[\left(\tfrac{2}{5}\cdot 1024-\tfrac{8}{3}\cdot 64\right)-\left(\tfrac{2}{5}\cdot 32-\tfrac{8}{3}\cdot 8\right)\right]$$

$$\pi\left(\tfrac{2}{5}(992)-\tfrac{8}{3}(56)\right)=\boxed{\frac{3712\pi}{15}}$$

(9) p. 645 #10

(a) $x = t^2;\ y = t^3$

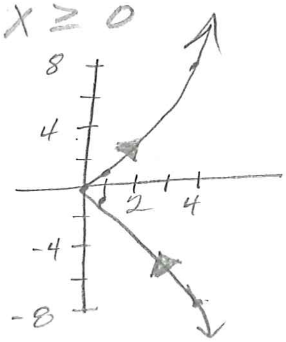
t	x	y
-2	4	-8
-1	1	-1
0	0	0
1	1	1
2	4	8

(b) $t = \sqrt[3]{y}$

$x = (\sqrt[3]{y})^2$

$\boxed{x = y^{2/3}}$

$t \in \mathbb{R}$
$y \in \mathbb{R}$
$x \geq 0$



(10) p. 655 #9 $\quad x = t^2 - t \quad y = t^2 + t + 1;\ (0, 3)$

$dx = 2t - 1 \quad dy = 2t + 1$

$$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} \rightarrow \frac{2t+1}{2t-1}$$

$t^2 - t = 0$

$t(t-1) = 0$

$\cancel{t=0},\ t = 1 \rightarrow y = 1^2 + 1 + 1$

$y = 3$

$$\frac{2(1)+1}{2(1)-1} = 3 \text{ (slope)}$$

$y - 3 = 3(x - 0) \longrightarrow \boxed{y = 3x + 3}$

(11) p. 655 #17 $\quad x = t^3 - 3t,\ y = t^2 - 3$ (horizontal tangent $f' = 0$) (vertical tangent f' DNE)

$\frac{dx}{dt} = 3t^2 - 3 \quad \frac{dy}{dt} = 2t$

$\frac{dy}{dx} = \frac{2t}{3t^2 - 3} \quad 2t = 0 \therefore t = 0 \quad (x, y) = \boxed{(0, -3) \text{ horizontal tangent}}$

$3t^2 - 3 = 3(t^2 - 1) = 3(t+1)(t-1) = 0 \quad t = 1 \text{ or } -1$

(x, y) at $t = 1$ $(-2, -2)$ $\quad (x, y)$ at $t = -1$ $(2, -2)$

$\boxed{(-2, -2) \text{ \& } (2, -2) \text{ vertical tangents}}$

(12) p. 666 #17 $\quad r = 5\cos\theta$

$r^2 = 5r\cos\theta$

$x^2 + y^2 = 5x$

$x^2 - 5x + y^2 = 0$

$x^2 - 5x + \frac{25}{4} + y^2 = \frac{25}{4}$ (Complete the square)

$(x - \frac{5}{2})^2 + y^2 = \frac{25}{4}$

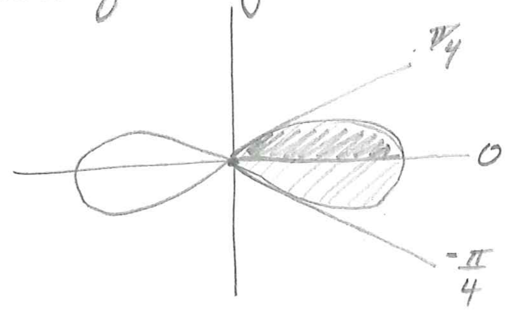
Circle with center @ $(5/2, 0)$ and radius of $5/2$.

(13) p. 668 # 69 $r = e^{\sin\theta} - 2\cos(4\theta)$

The parameter interval is $[0, 2\pi]$

$x\ [-3, 3]$ $y\ [-2.5, 3.5]$ Looks like a butterfly.

(14) p. 673 # 18 $r^2 = 4\cos(2\theta)$ passes through the pole when $r = 0$.



$4\cos(2\theta) = 0$

$\cos(2\theta) = 0$

$2\theta = \frac{\pi}{2} + \pi k$

$\theta = \frac{\pi}{4} + \frac{\pi}{2}k$

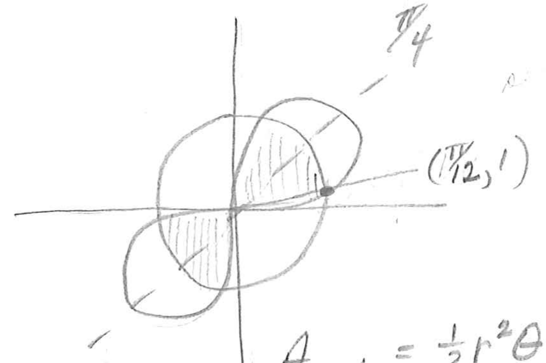
$$A = \int_{-\pi/4}^{\pi/4} \tfrac{1}{2}(4\cos(2\theta)\,d\theta = 2\int_0^{\pi/4} 2\cos(2\theta)\,d\theta$$

$u = \cos(2\theta)$

$du = 2\sin(2\theta)\,d\theta$

$$A = \Big[2\sin(2\theta)\Big]_0^{\pi/4} \qquad A = 2\big(\sin(\pi/2) - \underbrace{\sin(0)}_{0}\big) \qquad \boxed{A = 2}$$

(15) p. 673 # 33 $r^2 = 2\sin(2\theta)$ $r = 1$



The shaded region from 0 to π/4 is 1/4 shaded region.

$A_{sector} = \frac{1}{2}r^2\theta$

$2\sin(2\theta) = 1$ The area is separated

$\sin(2\theta) = \frac{1}{2}$ into 2 parts.

$2\theta = \frac{\pi}{6}$

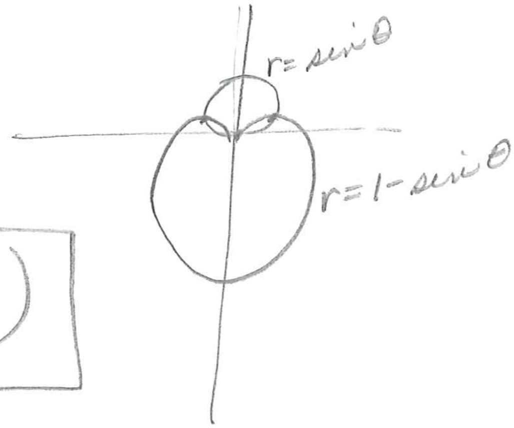
$\theta = \frac{\pi}{12}$

$$A = 4\int_0^{\pi/12} \tfrac{1}{2}(2\sin(2\theta)\,d\theta + 4\int_{\pi/12}^{\pi/4} \tfrac{1}{2}(1)^2\,d\theta$$

$$= \int_0^{\pi/12} 4\sin(2\theta)\,d\theta + \int_{\pi/12}^{\pi/4} 2\,d\theta = \Big[-2\cos(2\theta)\Big]_0^{\pi/12} + \Big[2\theta\Big]_{\pi/12}^{\pi/4}$$

$$(-\sqrt{3} + 2) - (\pi/2 - \pi/6) = \boxed{-\sqrt{3} + 2 + \pi/3}$$

(16) p. 673 # 37 $r = \sin\theta$, $r = 1 - \sin\theta$



$\sin\theta = 1 - \sin\theta$

$2\sin\theta = 1$

$\sin\theta = 1/2$

$\theta = \frac{\pi}{6}$ or $\frac{5\pi}{6}$ $\quad \boxed{(1/2, \pi/6), (1/2, 5\pi/6)}$

(17.) p. 673 # 47 $r = \theta^2$, $0 \le \theta \le 2\pi$ $\quad \frac{dr}{d\theta} = 2\theta$

$$L = \int_a^b \sqrt{r^2 + \left(\frac{dr}{d\theta}\right)^2}\, d\theta$$

$$L = \int_0^{2\pi} \sqrt{(\theta^2)^2 + (2\theta)^2}\, d\theta = \int_0^{2\pi} \sqrt{\theta^4 + 4\theta^2}\, d\theta = \int_0^{2\pi} \sqrt{\theta^2(\theta^2+4)}\, d\theta$$

$$L = \int_0^{2\pi} \theta\sqrt{\theta^2+4}\, d\theta \begin{bmatrix} u = \theta^2 + 4 \\ du = 2\theta\, d\theta \\ \frac{1}{2}du = \theta\, d\theta \end{bmatrix} \overset{s}{=} \int_4^{4\pi^2+4} \sqrt{u} \cdot \frac{1}{2} du = \frac{1}{2}\int_4^{4\pi^2+4} u^{1/2}\, du =$$

$$L = \frac{1}{2}\left[\frac{2}{3}u^{3/2}\right]_4^{4(\pi^2+1)} = \frac{1}{3}\left[4^{3/2}(\pi^2+1)^{3/2} - 4^{3/2}\right] = \frac{4^{3/2}}{3}\left[(\pi^2+1)^{3/2} - 1\right] =$$

$$L = \boxed{\frac{8}{3}\left[(\pi^2-1)^{3/2} - 1\right]}$$