

MAC 1140 FINAL EXAM REVIEW

There will be one page on the final where you will not be able to use your graphing calculator. Do the following problems #1 – 12 without your calculator:

1) Sketch the graph of $y = 3x^2 - 6x - 24$. State vertex, x and y intercepts, domain, range, and axis of symmetry.

2) Sketch the graph of $y = \frac{2x^2 - 2}{x^2 - x - 6}$. State asymptotes, x and y intercepts, horizontal asymptote intercept, domain and range.

3) Solve the inequality $\frac{2x^2 - 2}{x^2 - x - 6} \geq 0$. *The graph in problem #2 should help.*

4) Use transformations to sketch the graph of $y = 3^{x-1} - 3$. State x and y intercepts, asymptote, domain and range.

5) Find $\lim_{x \rightarrow -\infty} (3^{x-1} - 3)$. *The graph in problem#4 should help.*

6) Use transformations to sketch the graph of $y = \log_2(x + 4)$. State x and y intercepts, asymptote, domain and range.

7) Find $\lim_{x \rightarrow -4^+} \log_2(x + 4)$. *The graph in problem #6 should help.*

8) Make a rough sketch of the function $y = x(x + 2)^2(x - 3)^3$ going through (or just touching) the x-axis correctly at the x-intercepts and having the correct “end behavior”.

9) Write the solution of the inequality: $x(x + 2)^2(x - 3)^3 \geq 0$. *The graph in problem #8 should help.*

10) Make a rough sketch of the function $y = x^5 + 5x^4 - 4x^3 - 20x^2$ going through (or just touching) the x-axis correctly at the x-intercepts and having the correct “end behavior”.

11) Evaluate: a) $\log_3\left(\frac{1}{9}\right)$ b) $\ln(e^5)$ c) $\log(1000)$
 d) $\ln(1)$ e) $\log_{\frac{1}{8}}(4)$ f) $e^{3\ln 2}$

12) Write as a single logarithm: $4 \ln(2x^3) - 2 \ln(xy) + 3 \ln(x^2 y^3)$

You can use your calculator to help you with the rest of the problems.

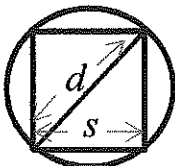
13) Given: $x^2 - y = 5$. Is y a function of x ? State domain and range.

14) Given: $f(x) = 4x^2 - x$ $g(x) = 3x + 1$

Find the following: a) $f(a+2)$ b) $(f+g)(x)$ c) $(f-g)(3)$ d) $(f \circ g)(x)$

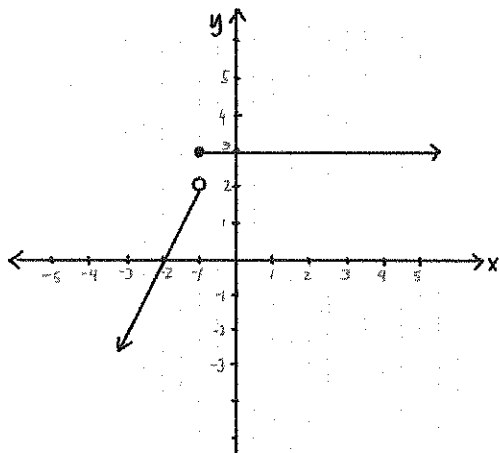
15) Find the difference quotients for the following functions:

a) $f(x) = 3x^2 - 4$ b) $g(x) = x^2 - 5x$ c) $F(x) = \frac{4}{x}$

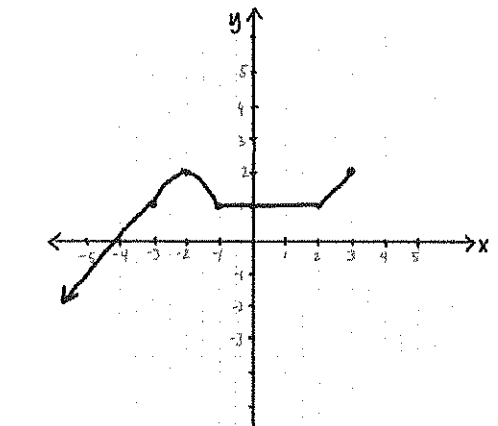
16)  a) Write the area of the circle as a function of d , the diameter of the circle.
b) Write s (the side of the square) as a function of d .

17) Graph $f(x) = \begin{cases} 2, & \text{for } x < -1 \\ 3x - 1, & \text{for } x \geq -1 \end{cases}$ 18) Graph $g(x) = \begin{cases} x^2, & \text{for } x \leq 1 \\ -x + 3, & \text{for } x > 1 \end{cases}$

- 19) Write the piecewise function that has this graph:



- 20) State intervals on which this function is increasing, decreasing, and constant:



- 21) Given: $f(x) = |x| - 3$
- Is the function even, odd, or neither?
 - Is it a one-to-one function?
- 22) $f = \{ (3, -2), (1, -4), (5, -4) \}$, $g = \{ (0, 2), (-2, 7), (3, 1), (1, 6), (-4, 8) \}$
- $f + g =$
 - $f \circ g =$
 - $g^{-1} =$
 - Is f invertible?
- 23) $f(x) = \frac{3x}{x-4}$, $f^{-1}(x) =$
- 24) $g(x) = \sqrt{x+3}$, $g^{-1}(x) =$
- 25) Write the exact solution(s) of $x^2 \geq 4x - 1$.
- 26) If a ball is thrown straight up with an initial velocity of 100 ft/sec from a height of 5 ft, what is the maximum height reached by the ball? How long does it take for the ball to hit the ground?
- 27) Joe wants to enclose a rectangular area for his chickens alongside his barn using 50 ft. of fencing. What dimensions will maximize the area fenced if the barn is used for one side of the rectangle?
- 28) State the quotient and remainder when dividing $2x^4 - 13x^2 + 2x + 1$ by $x + 3$.

29) Find all real and imaginary solutions of the following equations. (Give exact answers----no decimal approximations.)

a) $x^3 - x^2 + 2 = 0$

b) $x^4 - 4x^3 - 8x^2 + 32x - 21 = 0$

30) Write a 3rd degree polynomial equation that has these solutions: $-2, 3 \pm i$.

31) State the asymptotes for the following rational functions:

a) $y = \frac{5x^2 + 3}{x^2 - 9}$

b) $y = \frac{3x + 2}{x^2 + 1}$

c) $y = \frac{2x^2 - 3x + 1}{x - 2}$

32) Find the equation of the rational function that has:

VA: $x = 2$, HA: $y = 3$, x-intercept: $(4, 0)$

33) Solve the following equations. Round approximations to 2 decimal places.

a) $5(e^x) - 3 = 4$

b) $\sqrt{3} \cdot 3^{x+2} = 9$

c) $\log_3(x^2 - 5) = 2$

d) $\log x = 1 + \log(x - 3)$

e) $e^{2x} - 3e^x - 10 = 0$

f) $(\ln x)^2 + 2 \ln x - 8 = 0$

g) $\log_4 x + \log_4(2x + 5) = \log_4(3)$

34) What amount must you invest in order to have \$8000 in your account if compounded continuously at an annual interest rate of 4% for 5 years?

35) How much money will you have in your account at the end of 8 years if you invest \$2500 compounded monthly at an interest rate of 3.2%?

36) If an element has a half-life of 32 years and you have a 5 gram sample, how much will remain in 41 years? How long will it take for 1 gram to remain?

37) At 8am a bacteria colony contains 210 bacteria. If there are 500 bacteria at 11am, at what time will there be 750 bacteria? How many bacteria will there be at 1 pm?

38) Solve the following systems by substitution:

a) $y = 3x^2$

b) $y = 9^{x-1}$

c) $x^2 - 3y^2 = 13$

a) $y = 2x + 5$

b) $y = 3^{4x+3}$

c) $x - 3y = 1$

39) State the type of conic and write it in standard form:

a) $x^2 - 8x + y^2 + 14y + 40 = 0$

b) $2x^2 + 12x - y + 14 = 0$

(also state the center)

(also state the vertex)

40) State the center, foci, and asymptotes for: $\frac{(x-1)^2}{4} - \frac{(y+2)^2}{16} = 1$

41) State the equation of the parabola that has focus $(-3, 4)$ and directrix $x = 3$.

42) Write the equation of the hyperbola that has vertices $(-2, 5)$ & $(-2, 1)$ and foci $(-2, 6)$ & $(-2, 0)$.

43) If $\begin{vmatrix} 3 & -2 & -5 \\ x & 3 & 4 \\ -6 & 1 & 1 \end{vmatrix} = 3$, then what is x?

44) Solve by Gaussian Elimination: $\begin{cases} x - 2y - 3z = 4 \\ 2x - 4y + 5z = -3 \\ 5x - 6y + 4z = -7 \end{cases}$

45) Solve the same system (problem #44) by Cramer's Rule.

46) Find the solution of the system: $\begin{cases} 2x - 3y + z = 5 \\ 0x + y - 3z = 2 \\ 0x + 0y + 0z = 0 \end{cases}$

47) Set up the following as if you were going to do the partial fraction decomposition. (don't actually decompose)

$$\frac{5x^2 + 2}{x(x+3)^2(x^2 + 5)}$$

48) Decompose into partial fractions: $\frac{10x^2 - 9x + 3}{x^3 - 5x^2 + x - 5}$ (do the whole thing)

49) Given the sequence: 8, 6, 4, 2, 0, -2, ...

- Find a formula for the nth term. Do not write a recursion formula.
- Write a recursion formula for the sequence.
- Is the sequence Arithmetic, Geometric or neither?

50) a) Write the 1st four terms of the sequence: $a_n = \frac{(n+3)!}{2n}$

- Is the sequence Arithmetic, Geometric or neither?

51) Write the 15th term of the geometric sequence in which $r = \frac{1}{2}$ and $a_2 = 6$.

52) In an arithmetic sequence, if $a_1 = \frac{2}{3}$ and $a_5 = 0$, then what is a_8 ?

53) Write using summation notation with index i and starting with $i = 1$.

$$-\frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \frac{1}{9} - \frac{1}{11}$$

54) a) State the 1st four terms of the sequence defined recursively by
 $a_1 = \sqrt{2}$, $a_n = \sqrt{3} \cdot a_{n-1}$ for $n \geq 2$ b) Is it Arithmetic, Geometric, or neither?

55) A pendulum bob swings through an arc of 56 inches on its 1st swing and $\frac{3}{4}$ as far on each swing thereafter. How far will the bob travel altogether before coming to a complete stop?

56) Find the sum of each series by using an appropriate formula:

a) $\sum_{i=2}^{10} 2^{i-2}$ b) $210 + 105 + 52.5 + 26.25 + \dots$ c) $310 + 308 + 306 + \dots + 220$

57) Use the Binomial Theorem to write the first four terms of: $(x^2 + 3y)^7$
(Simplify each term as much as possible.)