MAC 1140 **Section 3.5**
Polynomial Functions **class notes**

$y = ax + b$ — 1st Degree (or <u>Linear</u>) Function

$y = ax^2 + bx + c$ — 2nd Degree (or <u>Quadratic</u>) Function

$y = ax^3 + bx^2 + cx + d$ — 3rd Degree (or <u>Cubic</u>) Function

$y = ax^4 + bx^3 + cx^2 + dx + e$ — 4th Degree Polynomial Function

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in general we have:

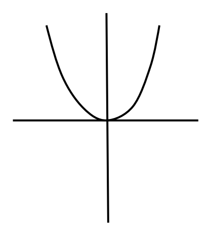
$$y = a_n x^n + a_{n-1}x^{n-1} + a_{n-2}x^{n-2} + \ . \ . \ . + a_2 x^2 + a_1 x + a_0$$

(this is an nth degree polynomial function)

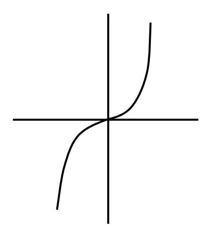
- A polynomial function of degree n has, <u>at most</u>, n-1 turning points.
- A polynomial function is a smooth, continuous curve….no breaks or sharp corners.

Power Functions (these are the simplest polynomial functions)

(even powers): $y = x^2$, $y = x^4$, $y = x^6$, $y = x^8$, etc. — these all have the "x^2 look":



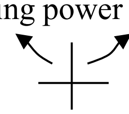
(odd powers): $y = x$, $y = x^3$, $y = x^5$, $y = x^7$, etc. — these all have the "x^3 look":

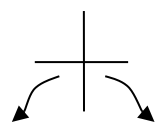


Transformations of basic power functions:

Graph: 1) $y=(x+2)^3$ 2) $y=(x-1)^4+3$ 3) $y=x^5-2$ 4) $y=-(x-3)^6+2$

- Polynomial functions in general ($y=a_nx^n+a_{n-1}x^{n-1}+\ .\ .\ .\ +a_1x+a_0$) have the same right and left "end behavior" as their corresponding power functions.

for n even: leading coefficient positive $\Rightarrow$  $as\ x\to\infty,\ y\to\infty$
$as\ x\to-\infty,\ y\to\infty$

leading coefficient negative $\Rightarrow$  $as\ x\to\infty,\ y\to-\infty$
$as\ x\to-\infty,\ y\to-\infty$

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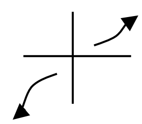
for n odd: leading coefficient positive $\Rightarrow$  $as\ x\to\infty,\ y\to\infty$
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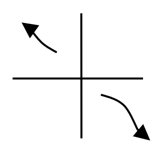
leading coefficient negative $\Rightarrow$  $as\ x\to\infty,\ y\to-\infty$
$as\ x\to-\infty,\ y\to\infty$

- Polynomial functions written in factored form:

If $(x-\text{k})$ is a factor of the function, then k is a zero of the function.
ex.: $y=(x-5)(2x+3)^2$ has zeros: 5 and –3/2
The graph of this function will have *x*-intercepts at 5 & -3/2

Factors of "odd multiplicity" (ex. $(x+5)^3$, $(2x-1)^7$, x, x^5) will <u>cross through</u> the *x*-axis at that *x*-intercept.

Factors of "even multiplicity" (ex. $(3x+2)^2$, x^4, $(x-7)^8$) will <u>just touch</u> the *x*-axis at that *x*-intercept.

To get a fairly accurate sketch of a polynomial function that is not a transformation of a basic power function, do the following:

a) Find the *x*-intercepts of the function by letting $y = 0$ and solving for *x* by factoring (you may also need to use the square root property).
*Note: We will solve higher degree equations that are not factorable in sections 3.3 and 3.4.

b) Determine if you will cross through or just touch the *x*-axis at the *x*-intercepts.

c) Find the *y*-intercept of the function by letting $x = 0$ and solving for *y*.

d) Keeping in mind the right and left end behavior of the function, connect your points with a nice, smooth curve.

e) Don't worry about getting the exact maximum or minimum values of your function. This is just a rough sketch.

f) Verify your graph with your graphing calculator.

Practice Exercises

Sketch the following by hand. Label all *x* and *y*-intercepts. Verify each graph with your graphing calculator.

1) $y = x^3 - 5x^2 + 6x$

2) $y = (x+2)^3(x-1)$

3) $y = x^4(x+1)^2$

4) $y = -(x-3)(x+1)^3$

5) $y = x^4 - 2x^3 - 8x^2$

6) $y = -x^3 + x^2 + 12x$

7) $y = x^4 + 5x^3 - x - 5$

8) $y = -(x+3)(x+6)(x-4)$

9) $y = (x-1)^2(x+3)^3$

10) $y = 3x^3 + 2x^2 - 3x - 2$