Topic 5: Define, Evaluate, and Compare Functions

Term	Meaning
Function	
Domain	
Range	
Relation	
Independent Variable	
Dependent Variable	
Linear Function	
Nonlinear Function	
Qualitative Graph	
Interval	

Pre-Chapter Review

Finding Slope from a Table

Find the slope from the table: Slope = ____________________

x	y
-2	5
0	9
2	13
4	17

Copy the (x,y) next to the table from the video.

Slope also equals: Change in____
Change in ____

$$\text{slope} = \frac{\text{change in } y}{\text{change in } x}$$

x	y
1	8
-3	5
-7	2
-11	-1

slope = $\frac{\text{change in } y}{\text{change in } x}$

x	y
0	-3
-2	0
-6	6
-8	9

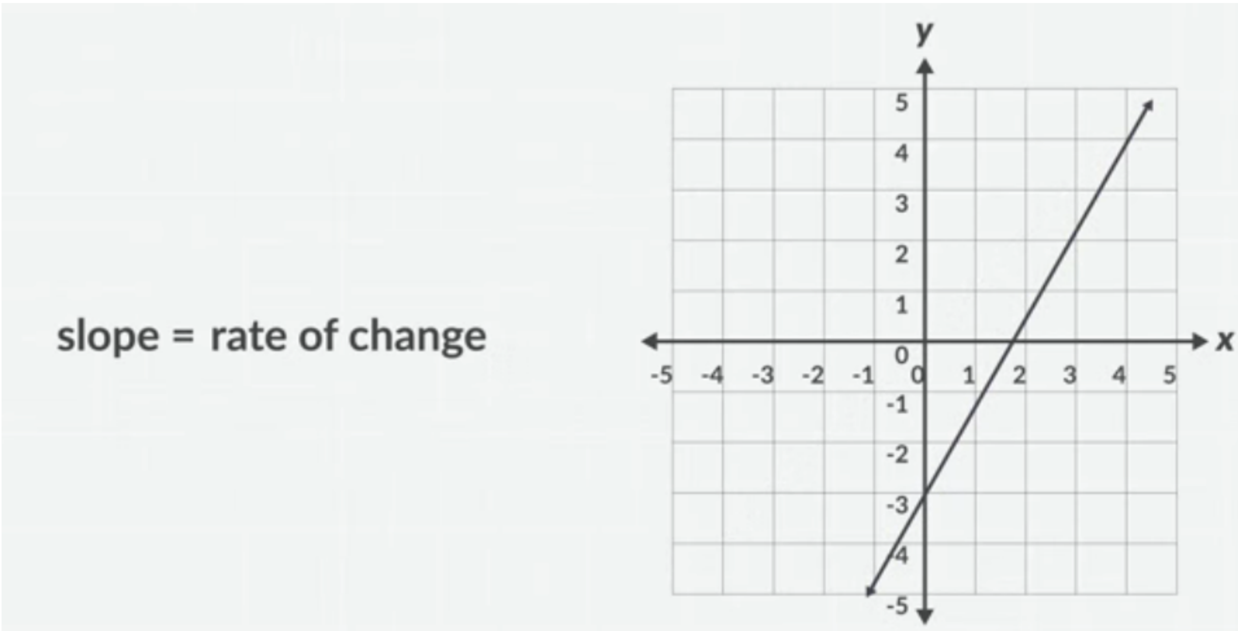
slope = $\frac{\text{change in } y}{\text{change in } x}$

x	y
21	9
16	7
11	5
6	3

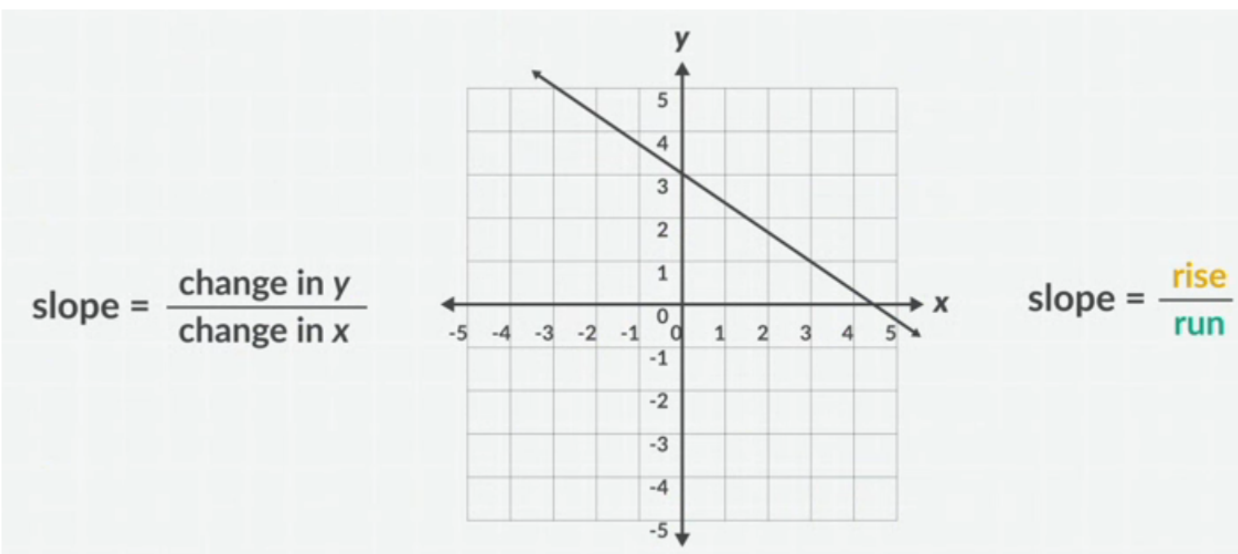
slope = $\frac{\text{change in } y}{\text{change in } x}$

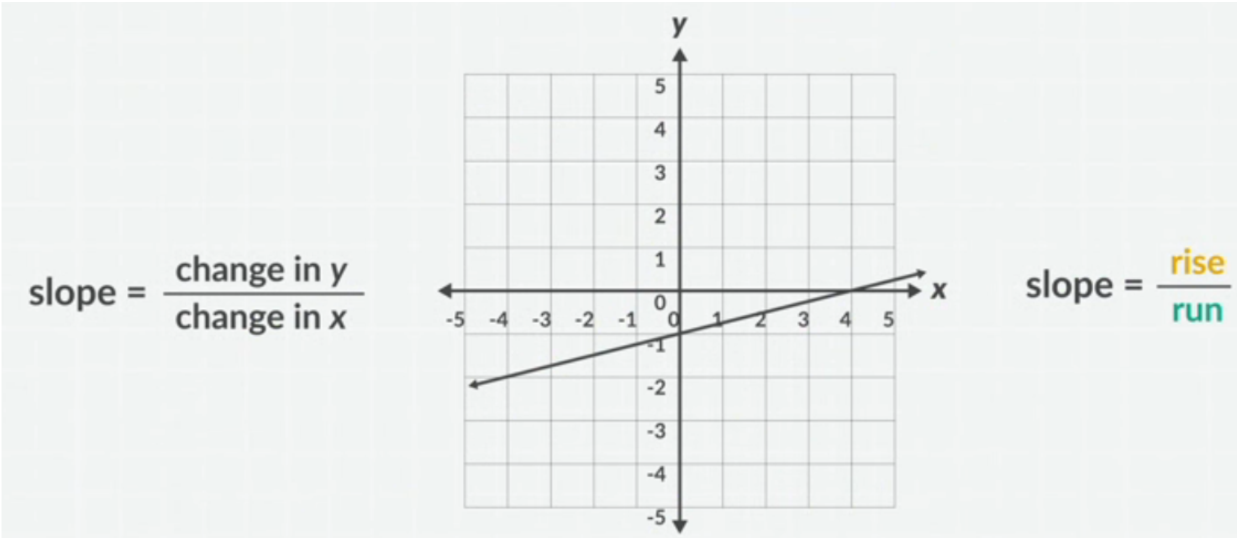
x	y
-3	13
1	11
9	7
13	5

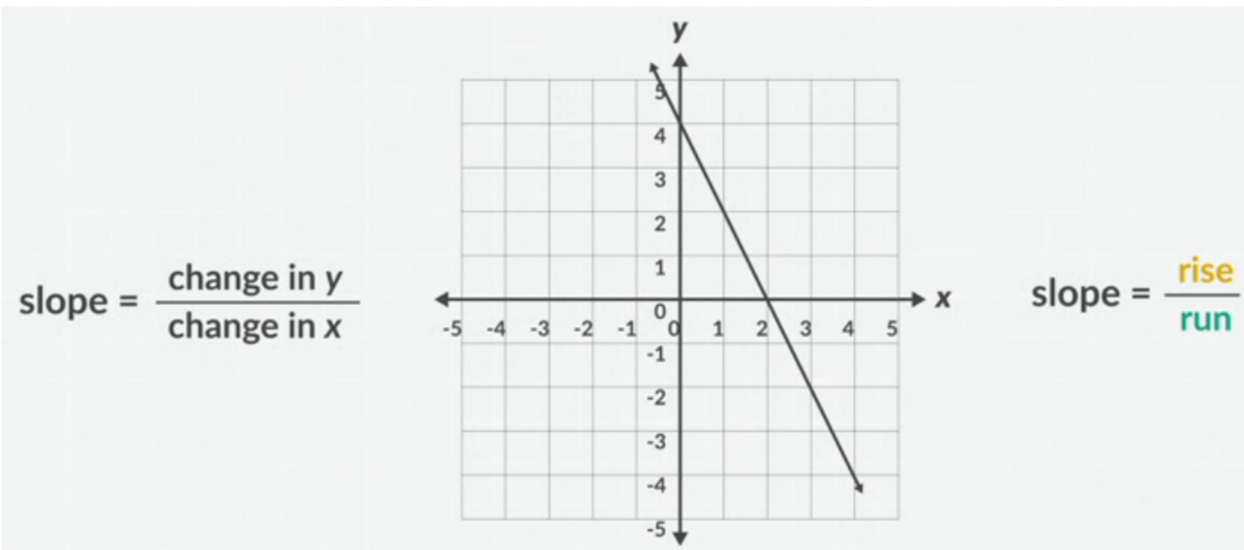
Find slope from Graphs:



Slope = <u>change in y</u> = ______
change in x = ______

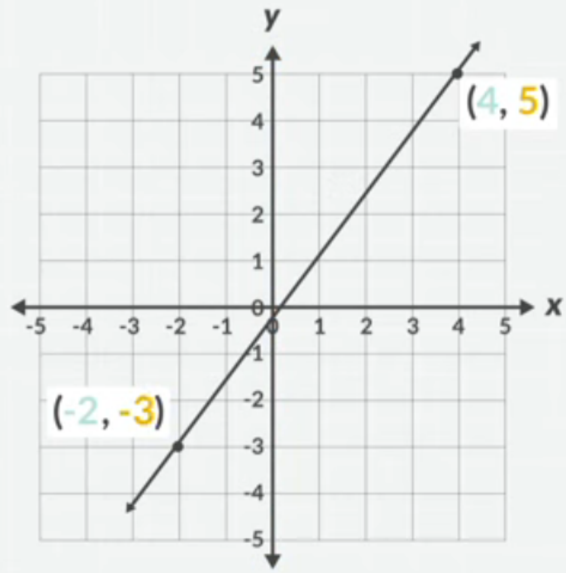



slope = change in y / change in x
y
x
5
4
3
2
1
0
-1
-2
-3
-4
-5
-5 -4 -3 -2 -1 0 1 2 3 4 5
slope = rise / run


slope = change in y / change in x
y
x
5
4
3
2
1
0
-1
-2
-3
-4
-5
-5 -4 -3 -2 -1 0 1 2 3 4 5
slope = rise / run

Finding Slope from Two Points

$$\text{slope} = \frac{\text{change in } y}{\text{change in } x}$$



(Formula) Slope = ________ Fill it in!

Find the slope of the line that passes between (-3 , -4) and (5 , -12)

(x_1, y_1) (x_2, y_2)

$$\text{slope} = \frac{y_2 - y_1}{x_2 - x_1}$$

Find the slope of the line that passes between (-4 , 3) and (-7 , 8)

$$\text{slope} = \frac{y_2 - y_1}{x_2 - x_1}$$

Find the slope of the line that passes between (-4 , -2) and (-6 , -12)

$$\text{slope} = \frac{y_2 - y_1}{x_2 - x_1}$$

All three videos review how to find the slope of linear equations - y = mx + b, where m is slope and b is the y-intercept. Linear equations or functions make graphs that are lines. Look at the graphs in each example from these 3 videos. They are ALL lines.

Lesson 1: Understand Relations & Functions

*Goal: Identify whether a **relation** is a **function***
*Identify the **domain and range** of a function*

A **relation** is a pairing of numbers in one set, called the **domain,** with numbers in another set, called the **range.** A relation is often represented as a set of ordered pairs (x, y). In this case, the domain is the set of x-values and the range is the set of y-values.

Function - a relation (set of ordered pairs) in which each element of the domain is paired with exactly one element of the range. In other words,
there is NO REPEATING OF X VALUES IN A FUNCTION!

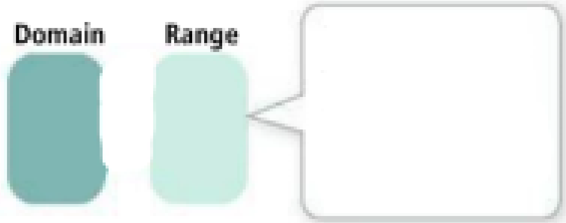
$\{(-2, 0.5), (0, 2.5), (4, 6.5), (5, 2.5)\}$
The domain is
The range is

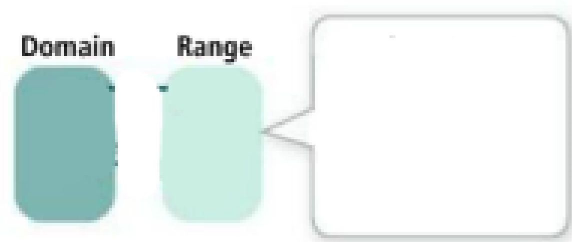
Problem 1 **Identifying Functions Using Mapping Diagrams**

Identify the domain and range of each relation. Represent the relation with a mapping diagram. Is the relation a function?

A $\{(-2, 0.5), (0, 2.5), (4, 6.5), (5, 2.5)\}$
The domain is $\{-2, 0, 4, 5\}$.
The range is $\{0.5, 2.5, 6.5\}$.

B $\{(6, 5), (4, 3), (6, 4), (5, 8)\}$
The domain is $\{4, 5, 6\}$.
The range is $\{3, 4, 5, 8\}$.





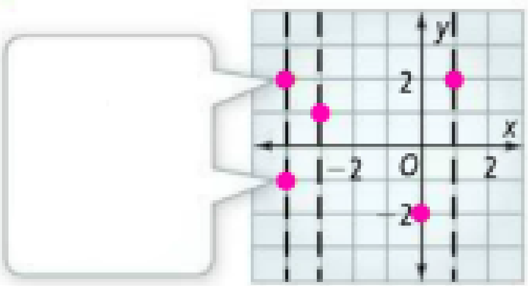
Got It? 1. Identify the domain and range of each relation. Represent the relation with a mapping diagram. Is the relation a function?

a. $\{(4.2, 1.5), (5, 2.2), (7, 4.8), (4.2, 0)\}$ b. $\{(-1, 1), (-2, 2), (4, -4), (7, -7)\}$

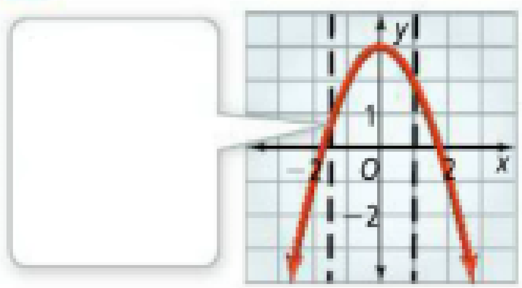
Problem 2 Identifying Functions Using the Vertical Line Test

Is the relation a function? Use the vertical line test.

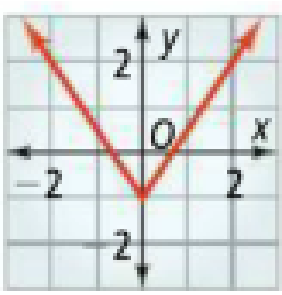
A $\{(-4, 2), (-3, 1), (0, -2), (-4, -1), (1, 2)\}$

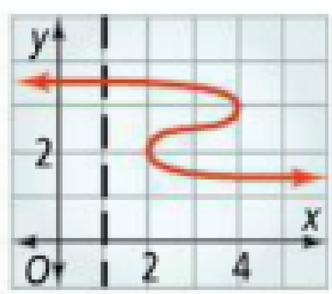


B $y = -x^2 + 3$



Try it: Are the graphs functions? Use the vertical line test to decide:





Extra Notes for in class previews:

A ________________ is any set of ordered pairs, and can be represented as a table or as a graph.

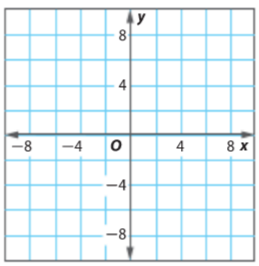
The ________________ of the relation is the set of x-coordinates.

The ________________ of the relation is the set of the y-coordinates.

A ________________ is a relation in which every member of the domain is paired with exactly **one** member of the range.

Express the relation $\{(-5, 2), (3, -1), (6, 2), (1, 7)\}$ as a table and a graph. Then state the domain and range.

x	y



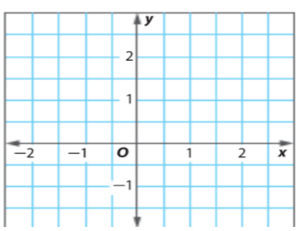
D:

R:

Function?

$\left\{\left(2\frac{1}{2}, -1\frac{1}{2}\right), \left(2, \frac{1}{2}\right), \left(-1, 2\frac{1}{2}\right), \left(-1, -1\frac{1}{2}\right)\right\}$

x	y



D:

R:

Function?

Independent Variable:

Dependent Variable:

The domain is the _____________ variable b/c it can be any value. The range is the

________________variable because it ______________________________.

Lesson 2: Analyze Functions

*Goal: Determine if a relation is a **linear function***
***Identify functions** by their equations, tables, and graphs*

Key Concept:

Identify Linear and Nonlinear Functions

In a previous lesson, you learned that linear functions have graphs that are straight lines. This is because the rate of change between any two data points is a constant. **Nonlinear functions** are functions whose rates of change are not constant. Therefore, their graphs are not straight lines.

<u>Draw the examples from the video here of NON-LINEAR FUNCTIONS:</u>

Determine whether each table represents a *linear* or *nonlinear* function. Explain.

1.

Change in x	x	y	Change in y
	2	50	
+2	4	35	−15
+2	6	20	−15
+2	8	5	−15

2.

Change in x	x	y	Change in y
	1	1	
+3	4	16	+15
+3	7	49	+33
+3	10	100	+51

<u>RATE CONSTANT?</u>

Got It? **Do this problem to find out.**

c. Tickets to the school dance cost $5 per student. Are the ticket sales a linear function of the number of tickets sold? Explain.

Number of Tickets Sold	1	2	3
Ticket Sales	$5	$10	$15

4. **A square has a side length of *s* inches. The area of the square is a function of the side length. Does this situation represent a linear or nonlinear function? Explain.**

DRAW THE TABLE AND GRAPH AND WRITE THE NOTES HERE:

Extra notes for class preview:

Use the ________________ of the graph of the function to determine if it is linear.

A nonlinear function has a graph that is not a straight line.

Does the table represent a linear or nonlinear function? Explain.

a.

+3 +3 +3

x	3	6	9	12
y	40	32	24	16

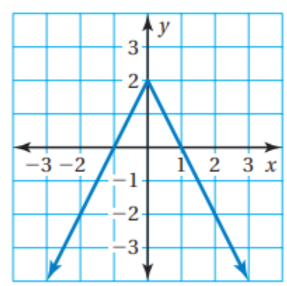
−8 −8 −8

As x increases by 3, y decreases by 8. The rate of change is constant. So, the function is linear.

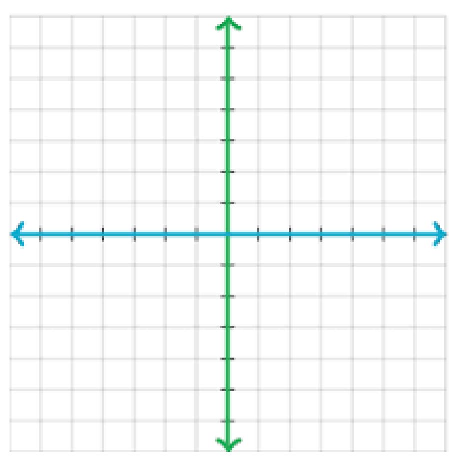
Does the table or graph represent a *linear* or *nonlinear* function? Explain.

x	y
0	25
7	20
14	15
21	10

x	y
2	8
4	4
6	0
8	−4



Check the ________ of ____________



Does the equation represent a *linear* or *nonlinear* function? Explain.

4. $y = x + 5$ **5.** $y = \frac{4x}{3}$ **6.** $y = 1 - x^2$

Lesson 3: Construct Functions to Model Linear Relationships

Goal: ***Write an equation*** *in the form y= mx+b to describe a linear function*

Explore It!

Erick wants to buy a new mountain bike that costs $250. He has already saved $120 and plans to save $20 each week from the money he earns for mowing lawns. He thinks he will have saved enough money after seven weeks.

Construct Functions to Model Linear Relationships

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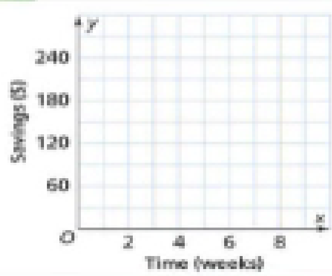
I can...
write an equation in the form $y = mx + b$ to describe a linear function.

A. Complete the table. Then graph the data.

Time (weeks)	0	1	2	3
Money Saved ($)	120			



B. How can you tell that the relationship is a linear function from the table? How can you tell from the graph?

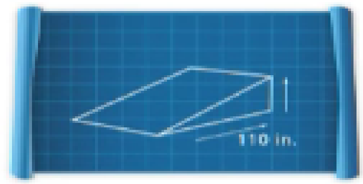
Focus on math practices

Generalize How can the different representations help you determine properties of functions?

EXAMPLE 1 Write a Function from a Graph

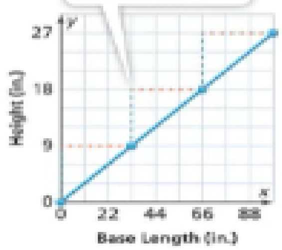
Scan for Multimedia

A plan for a skateboard ramp shows that the plywood for the triangular sides of the ramp should be cut such that for every 9 inches of height, the triangle should have a base that is 33 inches long. What is the height of the skateboard ramp shown?



STEP 1 Use a graph to represent the situation and to determine the slope.

For every 9 in. of height, the base is 33 in. long



STEP 2 Use the slope to write an equation that represents the function shown in the graph. Then use the equation to find the height for a base length of 110 inches.

Height | Slope | Base length

The equation is $y = \frac{3}{11}x$.

$y = \frac{3}{11}(110)$

$y = 30$

The height of the ramp is 30 inches.

The _____ of the line is the _____________ in __________ divided by the __________ in __________________ which is ___________________________. (***FILL IN THE BLANKS FROM THE SENTENCE UNDER STEP 1.)***

Try It!

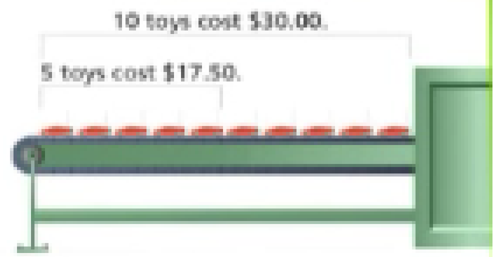
How will the height of the ramp change if the plan shows that for every 3 inches of height, the triangle should have a base that is 15 inches long?

Graph the function. The slope of the function shown in the graph is ☐. The equation of the function is $y =$ ☐ x. If the base length is 110 inches, then the height of the ramp will be ☐ inches.

Convince Me! Explain why the initial value and the y-intercept are equivalent.

EXAMPLE 2 Write a Function from Two Values

The cost to manufacture 5 toys is $17.50; the cost to manufacture 10 toys is $30. Construct a linear function in the form $y = mx + b$ that represents the relationship between the number of toys produced and the cost of producing them.



STEP 1 Determine the constant rate of change.

$$\frac{30 - 17.5}{10 - 5} = \frac{12.5}{5} = \frac{2.5}{1}$$

The constant rate of change is 2.5.

STEP 2 Use the slope and one set of values for x and y to find the y-intercept.

$30 = 2.5(10) + b \qquad 5 = b$

The initial value, or y-intercept, is 5.

The linear function that models this relationship is $y = 2.5x + 5$.

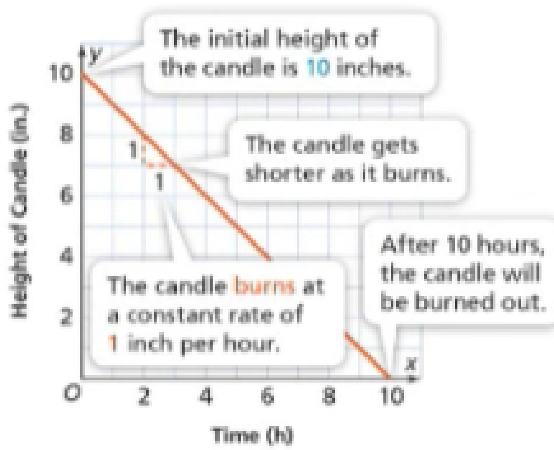
Try It!

Jin is tracking how much food he feeds his dogs each week. After 2 weeks, he has used $8\frac{1}{2}$ cups of dog food. After 5 weeks, he has used $21\frac{1}{4}$ cups. Construct a function in the form $y = mx + b$ to represent the amount of dog food used, y, after x weeks.

EXAMPLE 3 Interpret a Function from a Graph

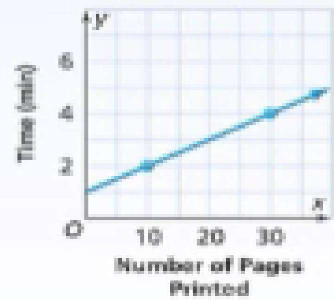
The graph shows the relationship of the height of a burning candle over time. What function represents the relationship?

The function $y = -1x + 10$ represents the relationship.



Try It!

The graph shows the relationship between the number of pages printed by a printer and the warm-up time before each printing. What function in the form $y = mx + b$ represents this relationship?



KEY CONCEPT

A function in the form $y = mx + b$ represents a linear relationship between two quantities, x and y.

Slope or constant rate of change

$$y = mx + b$$

y-intercept or initial value

2. **Make Sense and Persevere** Tonya is looking at a graph that shows a line drawn between two points with a slope of -5. One of the points is smudged and she cannot read it. The points as far as she can tell are (3, 5) and (x, 10). What must the value of x be? Explain.

3. **Reasoning** What is the initial value of all linear functions that show a proportional relationship?

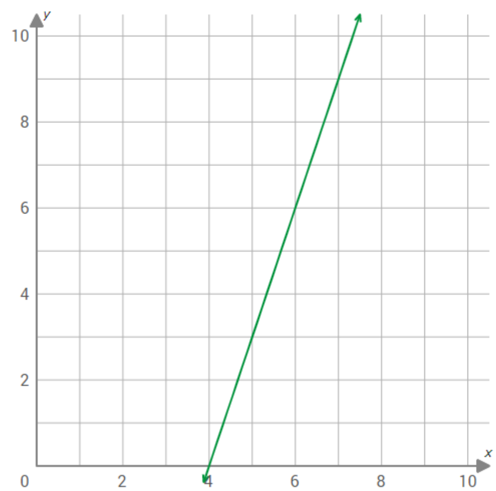
5. The data in the table below represent a linear relationship. Fill in the missing data.

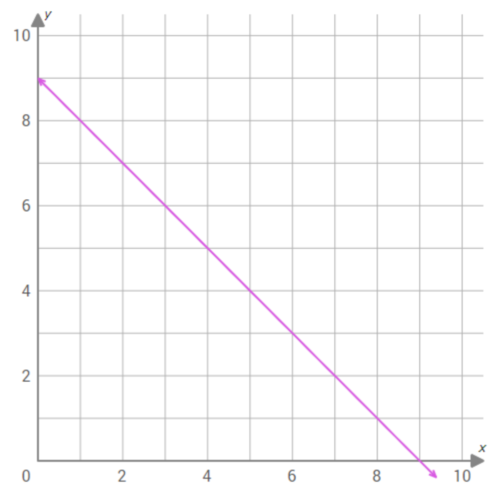
x	10	20		40
y	10	15	20	

6. What is an equation that represents the linear function described by the data in Item 5?

Extra Class Practice for Previews:

To write a linear function in slope intercept form, you need to identify the:



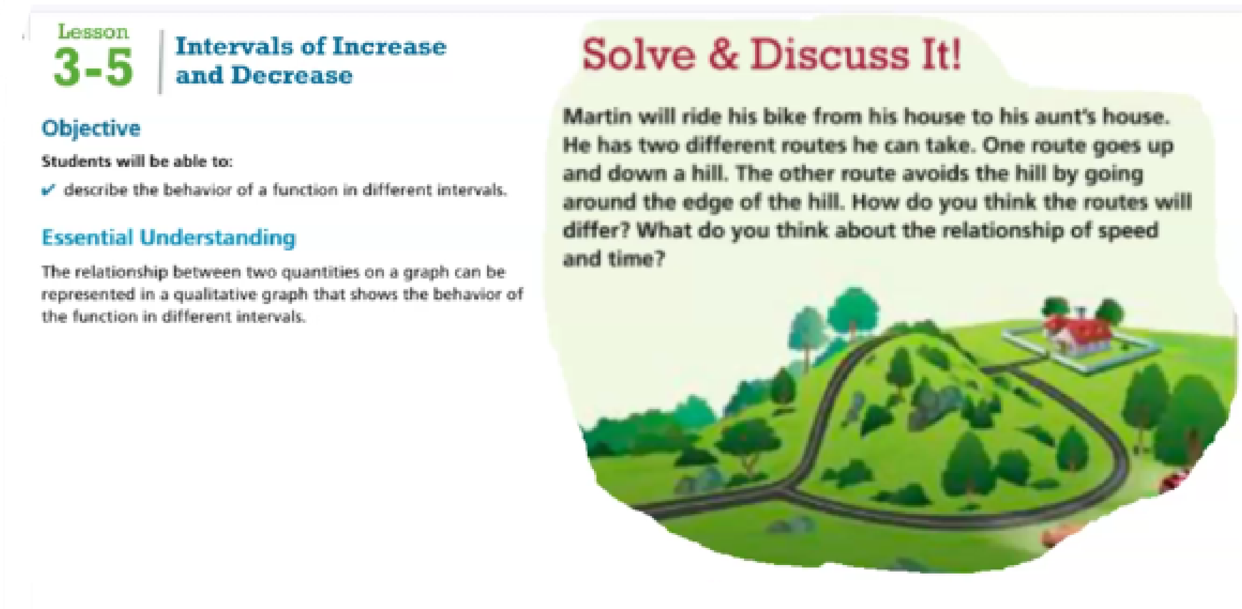


An airplane at 30,000 feet begins its descent. The plane descends 500 feet per minute. Identify the rate of change and initial value in this situation. Then, write an equation in the form of y=mx+b to model the height of the plane.

To find an initial value/
put the equation into Slope Intercept form and solve for the ____________________

Lesson 4: Intervals of Increase and Decrease

*Goal: describe the behavior of a function in different **intervals***


Lesson 3-5 | Intervals of Increase and Decrease

Objective

Students will be able to:

✔ describe the behavior of a function in different intervals.

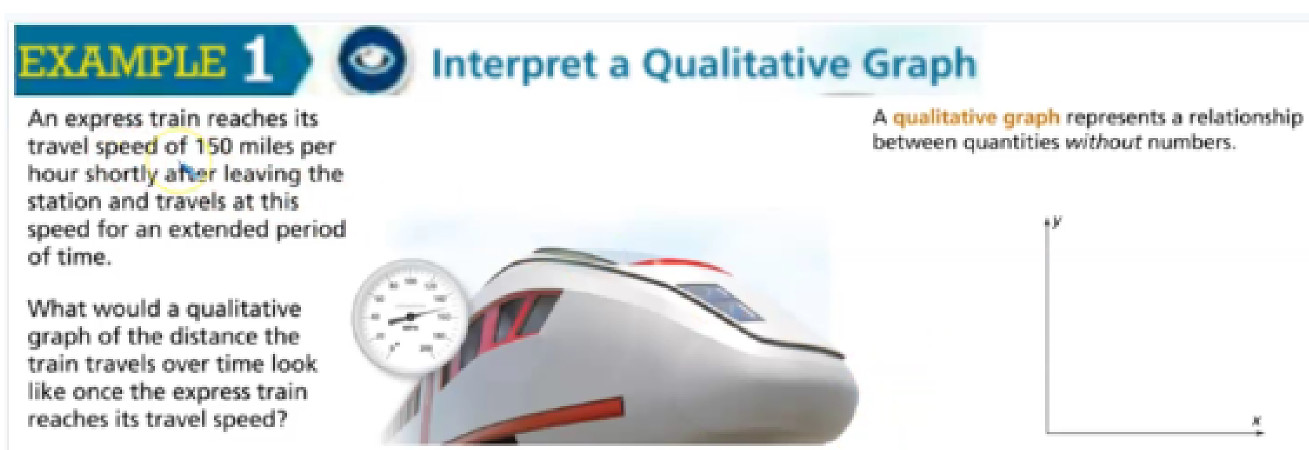
Essential Understanding

The relationship between two quantities on a graph can be represented in a qualitative graph that shows the behavior of the function in different intervals.

Solve & Discuss It!

Martin will ride his bike from his house to his aunt's house. He has two different routes he can take. One route goes up and down a hill. The other route avoids the hill by going around the edge of the hill. How do you think the routes will differ? What do you think about the relationship of speed and time?

<u>Be sure to draw the graphs and take notes from the video.</u>


EXAMPLE 1 Interpret a Qualitative Graph

An express train reaches its travel speed of 150 miles per hour shortly after leaving the station and travels at this speed for an extended period of time.

What would a qualitative graph of the distance the train travels over time look like once the express train reaches its travel speed?

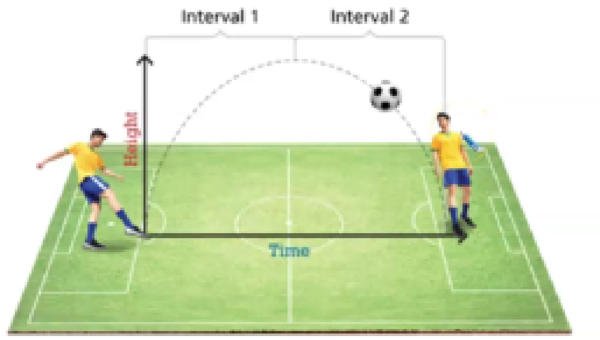
A qualitative graph represents a relationship between quantities *without* numbers.

<u>Be sure to draw the graphs and take notes from the video.</u>

EXAMPLE 2 Interpret the Graph of a Nonlinear Function

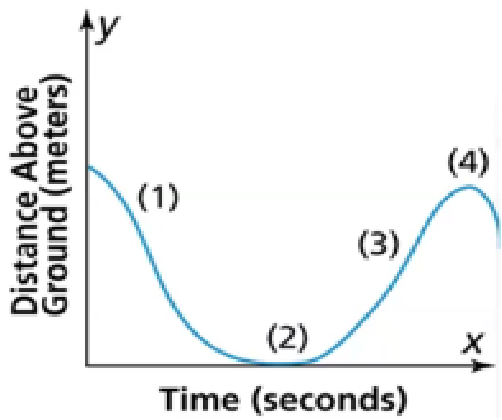
The graph shows the behavior of a ball that a soccer player kicks to a teammate. Describe how the height of the ball and time are related in each interval.

Determine whether the function is increasing, decreasing, or constant in each interval.



Try It!

The graph shows the behavior of Skylla skateboarding at a skateboard park. In which interval is the function increasing, decreasing, and constant?



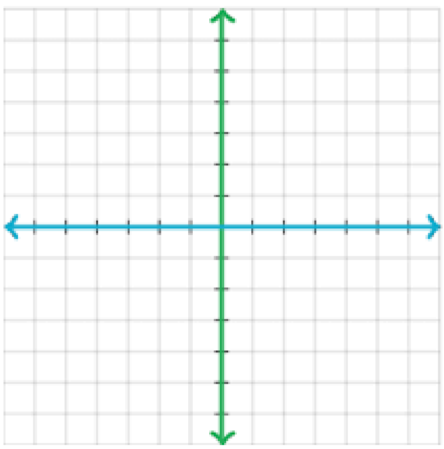
EXTRA CLASS NOTES FOR PREVIEW:

A ______________ is a graph used to represent situations that may not have numerical values or graphs in which numerical values are not included.

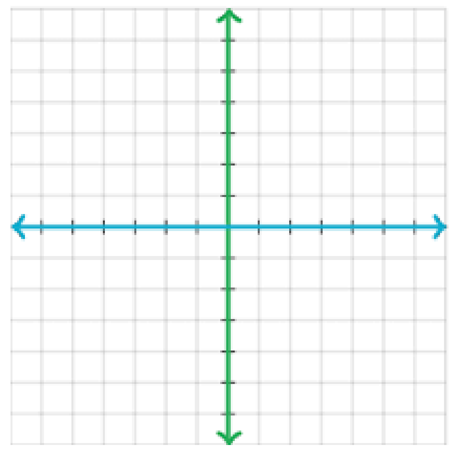
___________________ the period of time between two events or points in time.

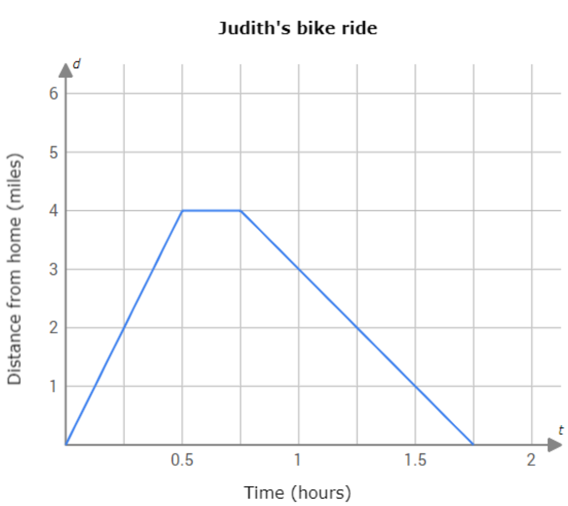
Functions can be_________________ or _________________ based on the __________.

Sketch a linear function that is increasing



Sketch a function that decreases then increases





What interval is Judith's distance decreasing?

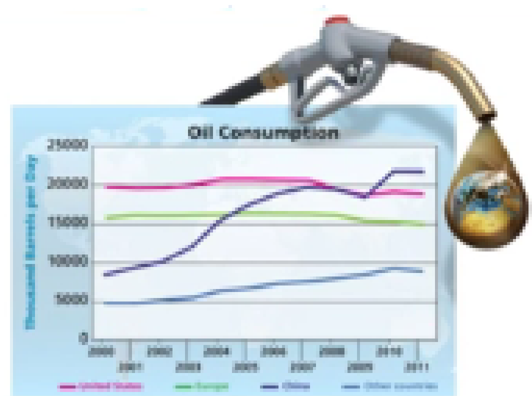
What interval is Judith's distance increasing?

What interval is Judith's distance constant?

Lesson 5: Sketch Functions from Descriptions

Goal: ***Draw a sketch of a graph*** *for a function that has been described*
Analyze and interpret *the sketch of a graph of a function*

The Environment Club is learning about oil consumption and energy conservation around the world. Jack says that the oil consumption in the United States has dropped a lot. Ashley says that the oil consumpion in China is the biggest problem facing the world environment.



A. Do you agree or disagree with Jack's statement? Construct an argument based on the graph to support your position.

<u>Write down whether to agree or disagree with Jack and Ashley.</u>

How does the sketch of a graph of a function help describe its behavior?

<u>Write down the answer to this question.</u>

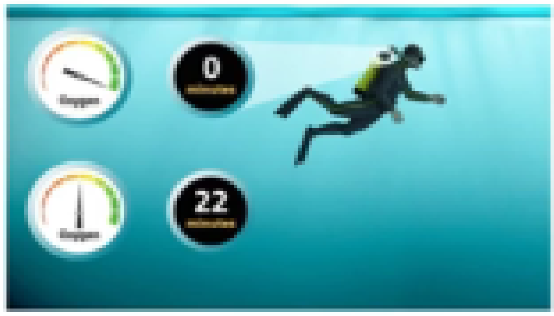
Sketch the Graph of a Linear Function

What does the graph of the function look like?

Step 1: Identify the two variables

Input variable: t (time)

Output variable: ℓ
(oxygen level in the tank)



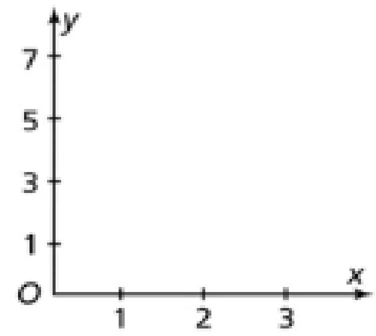
Step 2: Analyze the relationship between the two variables.

<u>Write down the analysis from the video:</u>

Step 3:

Sketch and label a graph that shows the behavior of the function.
When the dive begins, the oxygen tank is full.

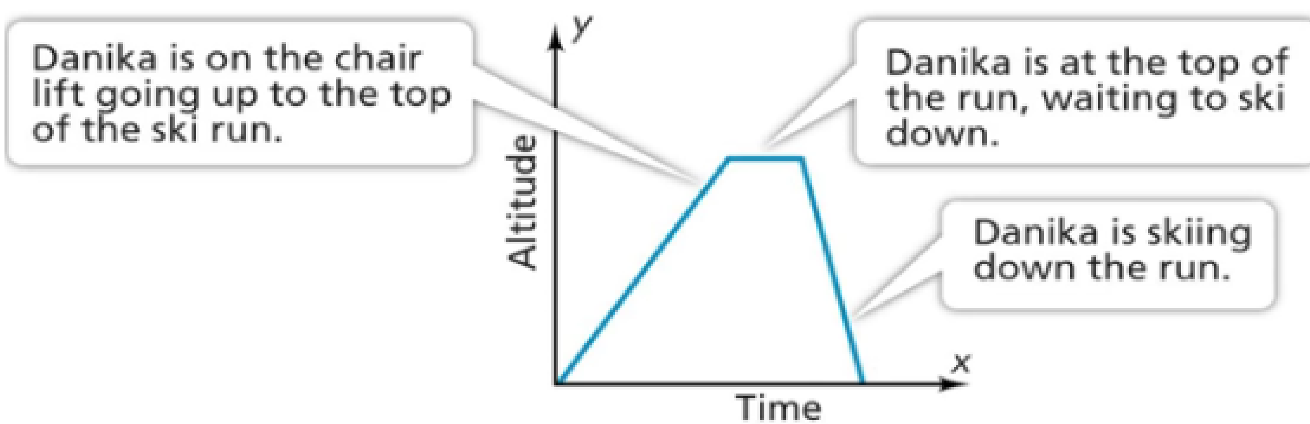
The weight of the water exerts pressure on a diver. At a depth of 10 feet, the water pressure is 19.1 pounds per square inch (psi) and at a depth of 14 feet, the water pressure is 20.9 psi. Complete the statements, then sketch the qualitative graph of this function.



The input, or x-variable, is ______.

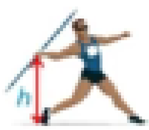
The output, or y-variable, is ______.

Danika sketched the relationship between altitude and time for one of her ski runs. Describe the behavior of the function in each interval.



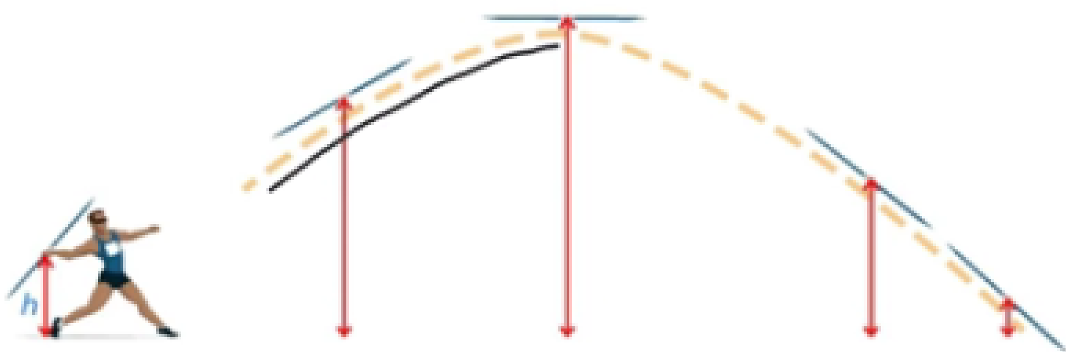
STEP 1 Identify the two variables in the relationship.

Input variable: ____________



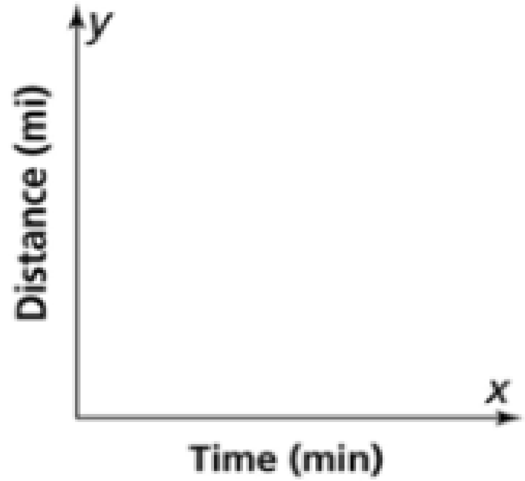
Output variable: ____________

STEP 2: Analyze the relationship between time and height.



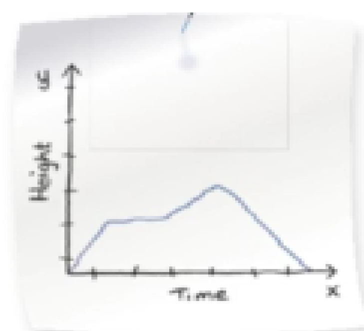
When José first throws the javelin, it increases in height. After it reaches its highest point, its height decreases until it hits the ground.

Jackson rides his bike from his home for 30 minutes at a fast pace. He stops to rest for 20 minutes, and then continues riding in the same direction at a slower pace for 30 more minutes. Sketch a graph of the relationship of Jackson's distance from home over time.



You can sketch a graph of a function to describe its behavior. When sketching a function, follow these steps:

1. Identify the two variables (input, output) that have a relationship.
2. Analyze the situation. Look for key words that indicate that the function is increasing, decreasing, or constant.
3. Sketch the graph.



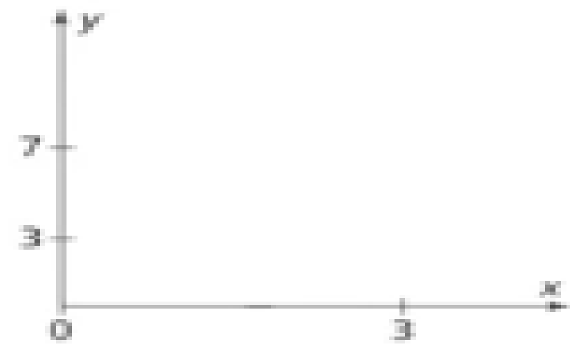
How do you know which variable goes with which axis when you graph?

<u>Be sure to answer this question here in your notes:</u>

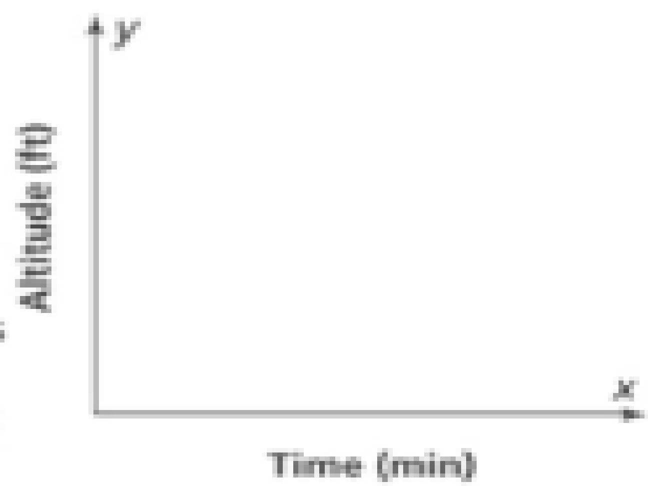
4. A class plants a tree. Sketch the graph of the height of the tree over time.

a. Identify the two variables.

c. Sketch the graph.

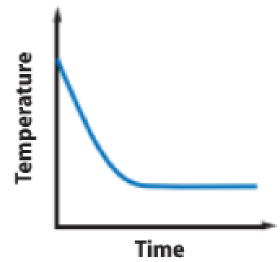


5. An airplane takes 15 minutes to reach its cruising altitude. The plane cruises at that altitude for 90 minutes, and then descends for 20 minutes before it lands. Sketch the graph of the height of the plane over time.

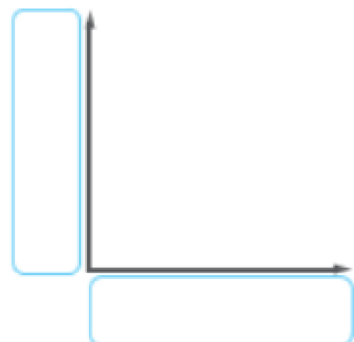


EXTRA EXAMPLES FOR CLASS PREVIEW:

The graph below displays the temperature of a cup of hot chocolate. Describe the change in the temperature over time.



A lion is resting when it sees another lion and races after it, picking up speed as it runs. Sketch a graph to represent the situation.



Emmy got a new puppy last year. The graph shows the puppy's weight over time. When the puppy was ______ months old he weighed 15 lbs. Over the next 3 months he grew from 15 to _______ lbs. Then he grew at a rate of _________ pounds per month for 2 months. After that…